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Service Specification and Matchmaking using Description Logic*

An Approach Based on Institutions

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Abstract. We propose a formal specification framework for functional aspects of services. We define services as operations which are specified by means of pre- and postconditions, for the specification of which we use extensions of description logic. The (extensions of) description logic and the specification framework itself are defined as institutions. This gives the framework a uniformity of definition and a solid algebraic and logical foundation. The framework can be used for the specification of service requests and service providers. Given a signature morphism from request to provider, we define when a service request is matched by a service provider, which can be used in service discovery. We provide a model-theoretic definition of matching and show that matching can be characterized by a semantic entailment relation which is formulated over a particular standard description logic. Thus proofs of matching can be reduced to standard reasoning in description logic for which one can use description logic reasoners.

1 Introduction

Service-oriented computing is emerging as a new paradigm based on autonomous, platform-independent computational entities, called *services*, that can be described, published, and dynamically discovered and assembled. An important part of a service is its public interface, which describes the service and should be independent of the technique used for implementing it. A service's interface can describe various aspects of the service, such as the service's location and communication protocols that can be used for interacting with the service.

In this paper, we confine ourselves to the investigation of those parts of a service's interface that describe the *functionality* offered to a service requester. Not all service specification approaches support this (see, e.g., WSDL [4]). Services that *are* endowed with such functional descriptions are often called *semantic web services* [18]. Semantic web services facilitate more effective (semi-)automatic service discovery and assembly, since the services' functional descriptions can be

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taken into account. In particular, such descriptions can be used for *matchmaking*, i.e., for finding a matching service provider for a particular service request.

Various techniques have been proposed for specifying semantic web services (see, e.g., [18, 19, 17, 12, 8, 22]). What most approaches have in common is that they suggest the use of *logical knowledge representation languages* for describing both service providers and service requests. Also, most approaches ([8] is an exception), including the approach we take in this paper, view semantic web services as *operations*, i.e., they can be invoked with some input, perform some computation and possibly return some output.

Where approaches for specifying semantic web services differ, is mostly the *kind* of knowledge representation language proposed, and the level of *formality*. In particular, in [12, 22], a formal service specification approach using first-order logic is presented, and in [18, 19] the use of so-called *semantic web markup languages* for service specification is proposed, but no formal specification language or semantics is defined. In this paper, we are interested in a formal approach to service specification, based on semantic web markup languages.

Semantic web markup languages are languages for describing the meaning of information on the web. The most widely used semantic web markup language is the Web Ontology Language (OWL) [21]. OWL is a family of knowledge representation languages that can be used for specifying and conceptualizing domains, describing the classes and relations between concepts in these domains. Such descriptions are generally called *ontologies* [9].

The formal underpinnings of the OWL language family are formed by *description logics* [1]. Description logics are formal ontology specification languages and form decidable fragments of first-order logic. Research on description logics has yielded sound and complete *reasoners* of increasing efficiency for various description logic variants (see [1] for more background). The fact that description logics come with such reasoners is an important advantage of using description logic for specifying services, since these reasoners can then be used for matchmaking.

In this paper, we propose a formal framework for specifying the functionality of services. Services are viewed as operations and we specify them using a particular description logic that corresponds to an expressive fragment of OWL, called OWL DL. As it turns out, we need to define several extensions of this description logic for its effective use in service specification. The formal tool that we use for defining the description logic, its extensions, and also the service specification framework itself, is *institutions* [7, 23]. The notion of an institution abstractly defines a logical system, viewed from a model-theoretic perspective. Institutions allow to define the description logics and the specification framework in a uniform and well-structured way.

In addition to defining a service specification framework, we also provide a model-theoretic definition of when a service request is *matched* by a service provider specification, and show that matching can be characterized by a semantic entailment relation which is formulated over our basic description logic. Proofs of matching can thus be reduced to standard reasoning in description logic, for which one can use description logic reasoners.

The organization of this paper is as follows. In Section 2, we define the description logic upon which we base our service specification framework. We informally describe the approach we take in this paper in some more detail in Section 3. Then, in Section 4, we define the extensions of the description logic of Section 2 that are needed for service specification, followed by the definition of the service specification framework in Section 5. The definition and characterization of matching are presented in Section 6, and we conclude the paper in Section 7.

2 The Description Logic $\mathcal{SHOIN}^+$

In this section, we present the description logic $\mathcal{SHOIN}^+$, on which we base our service specification framework. The logic $\mathcal{SHOIN}^+$ is based on $\mathcal{SHOIN}^+(\mathbf{D})$ [11]. $\mathcal{SHOIN}^+(\mathbf{D})$ is the logic $\mathcal{SHOIN}(\mathbf{D})$, extended with a particular construct that was needed in [11] to show that OWL DL ontology entailment can be reduced to knowledge base satisfiability in $\mathcal{SHOIN}(\mathbf{D})$. That construct also turns out to be useful for service specification. In this paper, we will omit datatypes and corresponding sentences from $\mathcal{SHOIN}^+(\mathbf{D})$ since it does not affect the essence of the presented ideas and would only complicate the presentation. This leaves us with the logic $\mathcal{SHOIN}^+$.

We will define $\mathcal{SHOIN}^+$ as an institution. Loosely speaking, an institution is a tuple $Inst = \langle Sig_{Inst}, Sen_{Inst}, Mod_{Inst}, \models_{Inst,\Sigma}\rangle$, where Sig_{Inst} is a category of signatures, Sen_{Inst} is a functor that yields for each signature from Sig_{Inst} a set of sentences, Mod_{Inst} is a functor yielding a category of models for each signature from Sig_{Inst}, and $\models_{Inst,\Sigma}$ for each signature $\Sigma \in |Sig_{Inst}|$ is a satisfaction relation specifying when a model of $|Mod_{Inst}(\Sigma)|$ satisfies a sentence of $Sen_{Inst}(\Sigma)$. Moreover, for each signature morphism $\sigma : \Sigma \to \Sigma'$, sentence $\phi \in Sen_{Inst}(\Sigma)$, and model $\mathcal{M}' \in |Mod_{Inst}(\Sigma')|$, the so-called satisfaction condition should hold: $\mathcal{M}' \models_{Inst,\Sigma'} \sigma(\phi) \Leftrightarrow \mathcal{M}'|_\sigma \models_{Inst,\Sigma} \phi$, where $\mathcal{M}'|_\sigma$ is the reduct of $\mathcal{M}'$ with respect to σ. For details, we refer to [7, 23]. For all institutions defined in this paper, the details, in particular model morphisms and the proof of the satisfaction condition, are provided in the appendix.

We now define the institution $\mathcal{SHOIN}^+ = \langle Sig_{\mathcal{S}^+}, Sen_{\mathcal{S}^+}, Mod_{\mathcal{S}^+}, \models_{\mathcal{S}^+,\Sigma}\rangle$. The definition is similar to the way OWL DL, the semantic web markup language corresponding to $\mathcal{SHOIN}(\mathbf{D})$, was defined as an institution in [15]. We illustrate our definitions using a running example of a service GA for making garage appointments, which allows to make an appointment with a garage within a given day interval. Such a service is part of the automotive case study of the SENSORIA project[1] on service-oriented computing.

The basic elements of $\mathcal{SHOIN}^+$ are concept names N_C, role names N_R, and individual names N_i, which together form a $\mathcal{SHOIN}^+$ signature $\langle N_C, N_R, N_i\rangle$. They are interpreted over a domain of elements called individuals. A concept name is interpreted as a set of individuals, a role name as a set of pairs of individuals, and an individual name as a single individual.

[1] `http://sensoria-ist.eu`

Definition 1 *($\mathcal{SHOIN}^+$ signatures: $Sig_{\mathcal{S}^+}$)* A $\mathcal{SHOIN}^+$ signature Σ is a tuple $\langle N_C, N_R, N_i \rangle$, where N_C is a set of concept names, $N_R = R \cup R^-$, where R is a set of (basic) role names and $R^- = \{r^- \mid r \in R\}$, is a set of role names, and N_i is a set of individual names. The sets N_C, N_R, and N_i are pairwise disjoint. A $\mathcal{SHOIN}^+$ signature morphism $\sigma_{\mathcal{S}^+} : \Sigma \rightarrow \Sigma'$ consists of a mapping of the concept names of Σ to concept names of Σ', and similarly for role names and individual names.

A simplified signature Σ^{GA} for our garage appointment service GA can be specified as follows: $N_C = \{\text{Appointment}, \text{Day}, \text{WDay}, \text{WEDay}, \text{Hour}, \text{String}\}$, $N_R = \{\text{after}, \text{before},\text{hasDay}, \text{hasHour}\}$, $N_i = \{1, 2, \ldots, 24, \text{mon}, \text{tue}, \ldots, \text{sun}\}$. The role names "after" and "before" will be used to express that a particular (week or weekend) day or hour comes before or after another day or hour, and "hasDay" and "hasHour" will be used to express that an appointment is made for a particular day and hour, respectively.

The main building blocks of $\mathcal{SHOIN}^+$ sentences are (composed) concepts, which can be constructed using concept names, individual names, and role names. For example, the concept $C_1 \sqcap C_2$ can be formed from the concepts C_1 and C_2, and is interpreted as the intersection of the interpretations of C_1 and C_2. Similarly, $C_1 \sqcup C_2$ denotes the union of the interpretations of C_1 and C_2. The concept $\exists r.C$ denotes all the individuals that are related to an individual from concept C over the role r, and several other composed concepts can be constructed.

Concepts, individual names, and role names are then used to construct sentences. For example, $C_1 \sqsubseteq C_2$ denotes that C_1 is a subconcept of C_2, and $a : C$ denotes that the individual represented by the individual name a belongs to concept C. The construct that $\mathcal{SHOIN}$ is extended with to form $\mathcal{SHOIN}^+$ is $\exists C$, which means that the interpretation of concept C is not empty. Definition 2 only contains those concepts and sentences that are used in the example. For a complete definition, we refer to Appendix A.

Definition 2 *($\mathcal{SHOIN}^+$ sentences: $Sen_{\mathcal{S}^+}$)* Let $\Sigma = \langle N_C, N_R, N_i \rangle \in |Sig_{\mathcal{S}^+}|$ be a $\mathcal{SHOIN}^+$ signature, and let $A \in N_C$, $r \in N_R$, and $a, a_1, a_2 \in N_i$. The sentences $Sen_{\mathcal{S}^+}(\Sigma)$ are then the axioms ϕ as defined below.

$$C ::= A \mid \top \mid \bot \mid \neg C \mid C_1 \sqcap C_2 \mid C_1 \sqcup C_2 \mid \{a\} \mid \exists r.C \mid \forall r.C$$
$$\phi ::= C_1 \sqsubseteq C_2 \mid r_1 \sqsubseteq r_2 \mid a : C \mid r(a_1, a_2) \mid \exists C$$

A $\mathcal{SHOIN}^+$ model or interpretation $\mathcal{I}$ is a pair $\langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$ where $\Delta^{\mathcal{I}}$ is a domain of individuals, and $\cdot^{\mathcal{I}}$ is an interpretation function interpreting concept names, role names, and individual names over the domain.

Definition 3 *($\mathcal{SHOIN}^+$ models: $Mod_{\mathcal{S}^+}$)* Let $\Sigma = \langle N_C, N_R, N_i \rangle \in |Sig_{\mathcal{S}^+}|$ be a $\mathcal{SHOIN}^+$ signature, where $N_R = R \cup R^-$ as specified in Definition 1. A model (or interpretation) $\mathcal{I}$ for $\mathcal{SHOIN}^+$ is a pair $(\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ consisting of a non-empty domain $\Delta^{\mathcal{I}}$ of individuals and an interpretation function $\cdot^{\mathcal{I}}$ which maps each concept name $A \in N_C$ to a subset $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$, each basic role name

$r \in R$ to a binary relation $r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$, and each individual name $a \in N_i$ to an element $a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$. The interpretation of an inverse role $r^- \in R^-$ is $(r^-)^{\mathcal{I}} = \{(y,x) \mid (x,y) \in r^{\mathcal{I}}\}$.

The $\mathcal{SHOIN}^+$ satisfaction relation is defined by first defining the interpretation of composed concepts, and then defining when an interpretation satisfies a sentence.

Definition 4 *($\mathcal{SHOIN}^+$ satisfaction relation: $\models_{\mathcal{S}^+,\Sigma}$)* Let $\Sigma \in |Sig_{\mathcal{S}^+}|$ be a $\mathcal{SHOIN}^+$ signature and let $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}}) \in |Mod_{\mathcal{S}^+}(\Sigma)|$ be a Σ-model. The satisfaction relation $\models_{\mathcal{S}^+,\Sigma}$ is then defined as follows, and is lifted to sets of sentences in the usual way.

$$\begin{array}{llll}
\top^{\mathcal{I}} &= \Delta^{\mathcal{I}} & (C_1 \sqcap C_2)^{\mathcal{I}} &= C_1^{\mathcal{I}} \cap C_2^{\mathcal{I}} \\
\bot^{\mathcal{I}} &= \emptyset & (C_1 \sqcup C_2)^{\mathcal{I}} &= C_1^{\mathcal{I}} \cup C_2^{\mathcal{I}} \\
(\neg C)^{\mathcal{I}} &= \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}} & \exists r.C^{\mathcal{I}} &= \{x \in \Delta^{\mathcal{I}} \mid \exists y : (x,y) \in r^{\mathcal{I}} \text{ and } y \in C^{\mathcal{I}}\} \\
\{a\}^{\mathcal{I}} &= \{a^{\mathcal{I}}\} & \forall r.C^{\mathcal{I}} &= \{x \in \Delta^{\mathcal{I}} \mid \forall y : (x,y) \in r^{\mathcal{I}} \Rightarrow y \in C^{\mathcal{I}}\}
\end{array}$$

$$\begin{array}{llll}
\mathcal{I} \models_{\mathcal{S}^+,\Sigma} C_1 \sqsubseteq C_2 & \Leftrightarrow C_1^{\mathcal{I}} \subseteq C_2^{\mathcal{I}} & \mathcal{I} \models_{\mathcal{S}^+,\Sigma} r(a_1,a_2) & \Leftrightarrow (a_1^{\mathcal{I}}, a_2^{\mathcal{I}}) \in r^{\mathcal{I}} \\
\mathcal{I} \models_{\mathcal{S}^+,\Sigma} a : C & \Leftrightarrow a^{\mathcal{I}} \in C^{\mathcal{I}} & \mathcal{I} \models_{\mathcal{S}^+,\Sigma} \exists C & \Leftrightarrow \exists x : x \in C^{\mathcal{I}}
\end{array}$$

A set of description logic sentences can be used to specify relationships between concepts, and properties of individuals. Such a set of sentences is often called an ontology. We define an ontology formally as a so-called $\mathcal{SHOIN}^+$ presentation. Presentations over an arbitrary institution are defined as follows [23]. If $Inst = \langle Sig_{Inst}, Sen_{Inst}, Mod_{Inst}, \models_{Inst,\Sigma}\rangle$ is an institution where $\Sigma \in |Sig_{Inst}|$, then the pair $\langle \Sigma, \Phi\rangle$ where $\Phi \subseteq Sen_{Inst}(\Sigma)$ is called a presentation. A model of a presentation $\langle \Sigma, \Phi\rangle$ is a model $M \in |Mod_{Inst}(\Sigma)|$ such that $M \models_{Inst,\Sigma} \Phi$. Then $\mathbf{Mod}_{Inst}(\langle \Sigma, \Phi\rangle) \subseteq |Mod_{Inst}(\Sigma)|$ is the class of all models of $\langle \Sigma, \Phi\rangle$.

Definition 5 *($\mathcal{SHOIN}^+$ ontology)* A $\mathcal{SHOIN}^+$ ontology is a presentation $\langle \Sigma, \Omega\rangle$, where $\Sigma \in |Sig_{\mathcal{S}^+}|$ and $\Omega \subseteq Sen_{\mathcal{S}^+}(\Sigma)$. Its semantics is the class of Σ-models satisfying the axioms in Ω, i.e., $\mathbf{Mod}_{\mathcal{S}^+}(\langle \Sigma, \Omega\rangle)$.

Part of the ontology Ω^{GA} for our garage appointment service GA can be specified as follows, where the $\mathcal{SHOIN}^+$ signature is Σ^{GA} as defined above (we refer to Appendix E for the complete definition of the running example). The concept "∃hasDay.Day" consists of all individuals that are related to some individual of the concept "Day" over the role "hasDay". The axiom "∃hasDay.Day ⊑ Appointment" specifies that these individuals should belong to the concept "Appointment", i.e., only appointments can have a day associated to them. Here and in the following we use $C \equiv C'$ as a shorthand notation for $C \sqsubseteq C', C' \sqsubseteq C$ where C and C' are concepts.

$$\begin{array}{l}
\{\ \exists \mathrm{hasDay.Day} \sqsubseteq \mathrm{Appointment}, \exists \mathrm{hasHour.Hour} \sqsubseteq \mathrm{Appointment}, \exists \neg \mathrm{Appointment}, \\
\quad \mathrm{WDay} \sqcup \mathrm{WEDay} \equiv \mathrm{Day}, \mathrm{mon} : \mathrm{WDay}, \ldots, \mathrm{fri} : \mathrm{WDay}, \mathrm{sat} : \mathrm{WEDay}, \\
\quad \mathrm{sun} : \mathrm{WEDay}, 1 : \mathrm{Hour}, \ldots, 24 : \mathrm{Hour}, \mathrm{after(mon, mon)}, \mathrm{after(mon, tue)}, \ldots, \\
\quad \mathrm{after}(1,1), \mathrm{after}(1,2), \mathrm{after}(2,2), \mathrm{after}(1,3), \mathrm{after}(2,3) \ldots, \\
\quad \mathrm{before(mon, mon)}, \mathrm{before(tue, mon)}, \ldots\}
\end{array}$$

3 Overview of the Approach

The description logic $\mathcal{SHOIN}^+$ as defined in the previous section forms the basis for the specification of services in our framework. In this section, we present the general idea of how we propose to use $\mathcal{SHOIN}^+$ for the specification of services.

As in, e.g., [18, 19, 17, 12, 22], we see services as operations with input and output parameters that may change the state of the service provider if the service is called. In order to define the semantics of services, we thus need to represent which state changes occur if the service is called with a given input, and which output is returned. A semantic domain in which these aspects are conveniently represented are so-called labeled transition systems with output (LTSO), which are also used as a semantic domain for the interpretation of operations in [10, 3].

An LTSO consists, roughly speaking, of a set of states and a set of transitions between these states, labeled by the name of the operation (which is a service in our case) by which the transition is made, and the actual input and output parameters. In our setting, the states are $\mathcal{SHOIN}^+$ interpretations. That is, we represent a service provider state as a $\mathcal{SHOIN}^+$ interpretation, and interpret services as operating on these states. The actual inputs and outputs of services are interpretations of variables (treated here as individuals).

It is important to note that using $\mathcal{SHOIN}^+$ for service *specification* does not mean that the service provider needs to be *implemented* using $\mathcal{SHOIN}^+$. Techniques for implementing services and for describing the relation of its implementation with its specification are, however, outside the scope of this paper.

In our framework, states are thus $\mathcal{SHOIN}^+$ interpretations. The general idea is then that the pre- and postconditions of a service are specified in $\mathcal{SHOIN}^+$. However, in order to be able to express pre- and postconditions properly, we do not use $\mathcal{SHOIN}^+$ as it is, but define several extensions. That is, in the precondition one often wants to specify properties of the input of the service, and in the postcondition properties of the input and output of the service. For this, it should be possible to refer to the *variables* forming the formal input and output parameters of the service. However, $\mathcal{SHOIN}^+$ does not facilitate the use of variables. For this reason, we use an extension of $\mathcal{SHOIN}^+$ with variables, called $\mathcal{SHOIN}^+_{Var}$, where variables refer to individuals.

Moreover, in the postcondition one typically wants to specify how the state may change, i.e., to specify properties of a transition. Hence, we need to be able to refer to the source and target states of a transition. For this purpose, we define an extension of $\mathcal{SHOIN}^+_{Var}$ called $\mathcal{SHOIN}^{+bi}_{Var}$ which allows both the use of variables and reference to the source and target states of a transition.

All necessary extensions of $\mathcal{SHOIN}^+$ are defined as institutions, and we define their semantics through a reduction to $\mathcal{SHOIN}^+$. This reduction allows us to use description logic reasoners for computing matches between a service request and service provider, which will be explained in more detail in Section 6. Although the extensions can be reduced to $\mathcal{SHOIN}^+$, we use the extensions rather than (an encoding in) $\mathcal{SHOIN}^+$ to let our approach be closer to the formalisms of [10, 3], and to our intuitive understanding of the semantics of services.

4 Extensions of $\mathcal{SHOIN}^+$

In this section, we present the extensions of $\mathcal{SHOIN}^+$ that we use for the specification of pre- and postconditions of services later on. We do not provide the complete definitions. Details can be found in Appendix B.

The first extension is $\mathcal{SHOIN}^+_{Var}$, which extends $\mathcal{SHOIN}^+$ with variables. A $\mathcal{SHOIN}^+_{Var}$ signature is a pair $\langle \Sigma, X\rangle$ where Σ is a $\mathcal{SHOIN}^+$ signature and X is a set of variables. Sentences of $\mathcal{SHOIN}^+_{Var}$ are then defined in terms of $\mathcal{SHOIN}^+$ sentences, by adding X to the individuals of Σ, which is a $\mathcal{SHOIN}^+$ signature denoted by Σ_X.

Models of a $\mathcal{SHOIN}^+_{Var}$ signature $\langle \Sigma, X\rangle$ are pairs $(\mathcal{I}, \rho)$, where $\mathcal{I}$ is a Σ-interpretation, and $\rho : X \rightarrow \Delta^{\mathcal{I}}$ is a valuation assigning individuals to the variables. The semantics of $\mathcal{SHOIN}^+_{Var}$ sentences is then defined in terms of the semantics of $\mathcal{SHOIN}^+$ sentences by constructing a $\mathcal{SHOIN}^+$ interpretation $\mathcal{I}_\rho$ from $(\mathcal{I}, \rho)$, in which variables are treated as individual names that are interpreted corresponding to ρ. A similar construction, in which variables are treated as part of the signature, can be found in the institution-independent generalization of quantification [5].

The second extension is $\mathcal{SHOIN}^{+bi}_{Var}$, which is an extension of $\mathcal{SHOIN}^+_{Var}$ and allows both variables and references to source and target states of a transition. The $\mathcal{SHOIN}^{+bi}_{Var}$ signatures are the $\mathcal{SHOIN}^+_{Var}$ signatures, but sentences of a signature $\langle \Sigma, X\rangle$ are defined in terms of the sentences of $\mathcal{SHOIN}^+$ by adding for each concept name A of Σ a concept name $A@pre$, and similarly for role names.

Models are triples $(\mathcal{I}_1, \mathcal{I}_2, \rho)$, where $\mathcal{I}_1$ and $\mathcal{I}_2$ are $\mathcal{SHOIN}^+$ interpretations and ρ is a valuation. We require that the domains and the interpretations of individual names are the same in $\mathcal{I}_1$ and $\mathcal{I}_2$, i.e., individual names are constants. These restrictions are also typical for temporal description logics [16]. The idea of the semantics is then that a concept name $A@pre$ in a $\mathcal{SHOIN}^{+bi}_{Var}$ sentence refers to A in $\mathcal{I}_1$, and a concept name A refers to A in $\mathcal{I}_2$, and similarly for role names. On this basis we define the satisfaction relation by a reduction to $\mathcal{SHOIN}^+$.

Definition 6 *($\mathcal{SHOIN}^{+bi}_{Var}$ institution)* The institution $\mathcal{SHOIN}^{+bi}_{Var} = \langle Sig_{\mathcal{S}^{+bi}_{Var}}, Sen_{\mathcal{S}^{+bi}_{Var}}, Mod_{\mathcal{S}^{+bi}_{Var}}, \models_{\mathcal{S}^{+bi}_{Var},\Sigma}\rangle$ is defined as follows:

- The $\mathcal{SHOIN}^{+bi}_{Var}$ signatures are the $\mathcal{SHOIN}^+_{Var}$ signatures, $\langle \Sigma, X\rangle$, i.e., $Sig_{\mathcal{S}^{+bi}_{Var}} = Sig_{\mathcal{S}^{+}_{Var}}$.
- Let $\langle \Sigma, X\rangle$ be a $\mathcal{SHOIN}^{+bi}_{Var}$ signature. The $\mathcal{SHOIN}^{+bi}_{Var}$ sentences are then defined as $Sen_{\mathcal{S}^{+bi}_{Var}}(\langle \Sigma, X\rangle) \triangleq Sen^+_{\mathcal{S}}(\Sigma^{bi}_X)$ where Σ^{bi}_X is a $\mathcal{SHOIN}^+$ signature extending Σ_X (see above) by concepts names $A@pre$ for all concept names A in Σ and by role names $r@pre$ for all role names r in Σ.
- A $\mathcal{SHOIN}^{+bi}_{Var}$ model is a triple $(\mathcal{I}_1, \mathcal{I}_2, \rho)$ where $\mathcal{I}_1, \mathcal{I}_2 \in |Mod_{\mathcal{S}^+}(\Sigma)|$, $\mathcal{I}_1 = (\Delta^{\mathcal{I}_1}, \cdot^{\mathcal{I}_1})$, $\mathcal{I}_2 = (\Delta^{\mathcal{I}_2}, \cdot^{\mathcal{I}_2})$, $\Delta^{\mathcal{I}_1} = \Delta^{\mathcal{I}_2}$, and $a^{\mathcal{I}_1} = a^{\mathcal{I}_2}$ for all $a \in N_i$, and $\rho : X \rightarrow \Delta$ is a valuation where $\Delta \triangleq \Delta^{\mathcal{I}_1} (= \Delta^{\mathcal{I}_2})$.

– For each $\mathcal{SHOIN}^{+bi}_{Var}$ signature $\langle \Sigma, X\rangle \in |Sig_{\mathcal{S}^{+bi}_{Var}}|$, the satisfaction relation $\models_{\mathcal{S}^{+bi}_{Var},\langle \Sigma,X\rangle}$ is defined as follows by means of a reduction to $\models_{\mathcal{S}^{+},\Sigma^{bi}_X}$. Let $(\mathcal{I}_1, \mathcal{I}_2, \rho) \in Mod_{\mathcal{S}^{+bi}_{Var}}(\langle \Sigma, X\rangle)$ and let $\overset{\curlywedge}{\mathcal{I}}_\rho \in Mod_{\mathcal{S}^+}(\Sigma^{bi}_X)$ be defined as follows: $\Delta^{\overset{\curlywedge}{\mathcal{I}}_\rho} = \Delta^{\mathcal{I}_1} (= \Delta^{\mathcal{I}_2})$, $\cdot^{\overset{\curlywedge}{\mathcal{I}}_\rho} = \cdot^{(\mathcal{I}_2)_\rho}$ for concept names A, role names r, and individual names a of Σ, and $\cdot^{\overset{\curlywedge}{\mathcal{I}}_\rho} = \cdot^{(\mathcal{I}_1)_\rho}$ for concept names $A@pre$ and role names $r@pre$, where $(\mathcal{I}_1)_\rho$ and $(\mathcal{I}_2)_\rho$ are the extension of $\mathcal{I}_1$ and $\mathcal{I}_2$, respectively, to variables as defined above.

  We now define $(\mathcal{I}_1, \mathcal{I}_2, \rho) \models_{\mathcal{S}^{+bi}_{Var},\langle \Sigma,X\rangle} \phi \triangleq \overset{\curlywedge}{\mathcal{I}}_\rho \models_{\mathcal{S}^{+},\Sigma^{bi}_X} \phi$ for $\phi \in Sen_{\mathcal{S}^{+bi}_{Var}}(\langle \Sigma, X\rangle)$ and thus by definition also $\phi \in Sen_{\mathcal{S}^+}(\Sigma^{bi}_X)$.

5 Service Specification using Description Logic

Having defined suitable extensions of $\mathcal{SHOIN}^+$, we continue to define our service specification framework. The definitions are inspired by approaches for the formal specification of operations in the area of object-oriented specification [10, 3], although these approaches are not based on institutions.

In the context of semantic web services specified using description logics, services are generally assumed to operate within the context of an ontology (see, e.g., [8]). The ontology defines the domain in which the services operate by defining the relevant concepts and relations between them. Moreover, a service provider will often provide multiple services, which all operate in the context of the same ontology. We call a bundling of services together with an ontology a *service package*. We define a service as an operation that has a name and that may have input and output variables as follows.

Definition 7 *(service)* A service $serv = servName([X_{in}]) : [X_{out}]$ consists of a service name *servName*, and sequences of input and output variables $[X_{in}]$ and $[X_{out}]$, respectively, such that all x in $[X_{in}]$ and $[X_{out}]$ are distinct. We use $var_{in}(serv)$ and $var_{out}(serv)$ to denote the sets of input and output variables of *serv*, respectively.

A garage appointment service can be represented by $makeAppointment(name, from, to) : app$. This service takes a name of a client and two days in between which the appointment should be made, and returns the appointment that it has made.

Now, we formally define service packages as an institution, for which we need the following general preliminaries [23]. Let $Inst = \langle Sig_{Inst}, Sen_{Inst}, Mod_{Inst}, \models_{Inst,\Sigma}\rangle$ be an institution where $\Sigma \in |Sig_{Inst}|$. For any class $\mathcal{M} \subseteq |Mod_{Inst}(\Sigma)|$ of Σ-models, the theory of $\mathcal{M}$, $Th_\Sigma(\mathcal{M})$, is the set of all Σ-sentences satisfied by all Σ-models in $\mathcal{M}$, i.e., $Th_\Sigma(\mathcal{M}) = \{\phi \in Sen_{Inst}(\Sigma) \mid \mathcal{M} \models_{Inst,\Sigma} \phi\}$. The closure of a set Φ of Σ-sentences is the set $Cl_\Sigma(\Phi) = Th_\Sigma(\mathbf{Mod}_{Inst}(\Phi))$. A theory morphism $\sigma : \langle \Sigma, \Phi\rangle \to \langle \Sigma', \Phi'\rangle$ is a signature morphism $\sigma : \Sigma \to \Sigma'$ such that $\sigma(\phi) \in \Phi'$ for each $\phi \in \Phi$.

A service package signature Σ_{SP} is a pair $(\langle \Sigma, \Omega \rangle, Servs)$ where $\langle \Sigma, \Omega \rangle$ is a $\mathcal{SHOIN}^+$ ontology and *Servs* is a set of services. An *SP* signature morphism σ_{SP} from an *SP* signature Σ_{SP} to *SP* signature Σ'_{SP} then defines that there is a theory morphism from the ontology sentences of Σ_{SP} to those of Σ'_{SP}.

The sentences of an *SP* institution are used to specify the services and are of the form $\langle serv, \mathsf{pre}, \mathsf{post} \rangle$. Here, *serv* is the service that is being specified, and pre and post are the pre- and postconditions of the service, respectively. We now use the extensions of $\mathcal{SHOIN}^+$ as defined in Section 4 for the definition of pre and post. That is, the precondition is specified by means of $\mathcal{SHOIN}^+_{Var}$ sentences, where the variables that may be used are the variables of the input of *serv*. The postcondition is specified by means of $\mathcal{SHOIN}^{+bi}_{Var}$ sentences, which means that the postcondition can refer to the source and target states of a transition, and the variables that may be used are the variables of the input and output of *serv*.

The models of service packages are non-deterministic total labeled transition systems with output (see also Section 3). A transition system in our framework is a pair $\mathcal{T} = (Q, \delta)$. Q is the set of states, which are in our case $\mathcal{SHOIN}^+$ interpretations that satisfy the ontology of the service specification, i.e., the ontology is treated as an invariant that the specified service always fulfills. The set δ is the transitions between states. Each transition $t \in \delta$ has a source and a target state from Q. Furthermore, t is labeled with the service through which the transition is made, together with a valuation of the input variables of the service, expressing which are the actual input parameters of the service call. Any transition t is equipped with a valuation of the output variables, expressing which are the actual output parameters of the service call. Loosely speaking, a transition system $\mathcal{T} = (Q, \delta)$ satisfies a sentence $\langle serv, \mathsf{pre}, \mathsf{post} \rangle$, if for all transitions from an interpretation $\mathcal{I} \in Q$ to some $\mathcal{I}' \in Q$ through service *serv*, if pre holds in $\mathcal{I}$, then the transition to $\mathcal{I}'$ satisfies post.

Definition 8 *(service package (SP) institution)* The institution $SP = \langle Sig_{SP}, Sen_{SP}, Mod_{SP}, \models_{SP,(\langle \Sigma, \Omega \rangle, Servs)} \rangle$ is defined as follows:

- An *SP* signature is a pair $(\langle \Sigma, \Omega \rangle, Servs)$ where $\langle \Sigma, \Omega \rangle$ is a $\mathcal{SHOIN}^+$ ontology (see Definition 5), and *Servs* is a set of services. An *SP* signature morphism $\sigma_{SP} : (\langle \Sigma, \Omega \rangle, Servs) \rightarrow (\langle \Sigma', \Omega' \rangle, Servs')$ consists of a theory morphism $\sigma_\Omega : \langle \Sigma, Cl_\Sigma(\Omega) \rangle \rightarrow \langle \Sigma', Cl_{\Sigma'}(\Omega') \rangle$, and a mapping of each service $serv \in Servs$ to a service $serv' \in Servs'$, such that for each mapping from *serv* to $serv'$ it holds that *serv* and $serv'$ have the same number of input variables and the same number of output variables.
- An *SP* sentence is a triple $\langle serv, \mathsf{pre}, \mathsf{post} \rangle$, where *serv* is a service, and $\mathsf{pre} \subseteq Sen_{\mathcal{S}^+_{Var}}(\langle \Sigma, X_{in} \rangle)$, $\mathsf{post} \subseteq Sen_{\mathcal{SHOIN}^{+bi}_{Var}}(\langle \Sigma, X_{in,out} \rangle)$, where here and in the following $X_{in} = var_{in}(serv)$, $X_{out} = var_{out}(serv)$, and $X_{in,out} = var_{in}(serv) \cup var_{out}(serv)$.
- An *SP* model for this signature is a non-deterministic total labeled transition system with outputs $\mathcal{T} = (Q, \delta)$, where $Q \subseteq \mathbf{Mod}_{\mathcal{S}^+}(\langle \Sigma, \Omega \rangle)$ is a set of states and δ is a set of transitions between states, defined as follows. Let $Label = \{(serv, \rho_{in}) \mid serv \in Servs, \rho_{in} : var_{in}(serv) \rightarrow \Delta\}$, where $\Delta =$

$\bigcup\{\Delta^{\mathcal{I}} \mid \mathcal{I} \in Q\}$ and let *Output* be the set of valuations $\rho_{out} : X \rightarrow \Delta$ where X is an arbitrary set of variables. Then $\delta \subseteq Q \times \mathit{Label} \times (Q \times \mathit{Output})$ such that for all $(\mathcal{I}, (serv, \rho_{in}), (\mathcal{I}', \rho_{out})) \in \delta$ we have $\rho_{in} : var_{in}(serv) \rightarrow \Delta^{\mathcal{I}}$ and $\rho_{out} : var_{out}(serv) \rightarrow \Delta^{\mathcal{I}'}$, and $\mathcal{T}$ is total, i.e., for all $\mathcal{I} \in Q$ it holds that for all $l \in \mathit{Label}$ there is an $\mathcal{I}', \rho_{out}$ such that $(\mathcal{I}, (serv, \rho_{in}), (\mathcal{I}', \rho_{out})) \in \delta$.

  The reduct $\mathcal{T}'|_{\sigma_{SP}}$ where $\mathcal{T}' = (Q', \delta')$ is $(Q'|_{\sigma_{Ont}}, \delta'|_{\sigma_{SP}})$, where $Q'|_{\sigma_{Ont}} = \{\mathcal{I}'|_{\sigma_{Ont}} \mid \mathcal{I}' \in Q'\}$, and $\delta'|_{\sigma_{SP}}$ are all transitions $(\mathcal{I}_1|_{\sigma_{Ont}}, (serv, \rho_{in}|_{\sigma_{\mathcal{S}^+_{Var}}}), \mathcal{I}_2|_{\sigma_{Ont}}, \rho_{out}|_{\sigma_{\mathcal{S}^+_{Var}}})$ such that there is a transition $(\mathcal{I}_1, (\sigma_{SP}(serv), \rho_{in}), \mathcal{I}_2, \rho_{out}) \in \delta'$.

- Let $\Sigma_{SP} = (\langle \Sigma, \Omega \rangle, \mathit{Servs})$ be an *SP* signature, and let $\mathcal{T} = (Q, \delta) \in \mathit{Mod}_{SP}((\langle \Sigma, \Omega \rangle, \mathit{Servs}))$. We define $\mathcal{T} \models_{SP, \Sigma_{SP}} \langle serv, \mathsf{pre}, \mathsf{post} \rangle$ iff for all $\mathcal{I} \in Q$ and for all $\rho_{in} : X_{in} \rightarrow \Delta^{\mathcal{I}}$, if $(\mathcal{I}, \rho_{in}) \models_{\mathcal{S}^+_{Var}, \langle \Sigma, X_{in} \rangle} \mathsf{pre}$ the following holds: for all $(\mathcal{I}, (serv, \rho_{in}), \mathcal{I}', \rho_{out}) \in \delta$ it holds that $(\mathcal{I}, \mathcal{I}', \rho_{in,out}) \models_{\mathcal{S}^{+bi}_{Var}, \langle \Sigma, X_{in,out} \rangle} \mathsf{post}$. We use $\rho_{in,out}$ to denote the merging of the two valuations ρ_{in} and ρ_{out} to one valuation in the obvious way.

We now define a service package specification as an *SP* presentation, i.e., it consists of an *SP* signature and a set of *SP* sentences, and its semantics is the class of all its models.

Definition 9 *(service package specification)* A service package specification is a presentation $\langle \Sigma_{SP}, \Psi_{SP} \rangle$ where $\Sigma_{SP} \in |\mathit{Sig}_{SP}|$ and $\Psi_{SP} \subseteq \mathit{Sen}_{SP}(\Sigma_{SP})$ such that for each $serv \in \mathit{Servs}$ where $\Sigma_{SP} = \langle \mathit{Ont}, \mathit{Servs} \rangle$ there is exactly one sentence of the form $\langle serv, \mathsf{pre}, \mathsf{post} \rangle$ in Ψ_{SP}. Its semantics is the class of Σ_{SP}-models satisfying the axioms in Ψ_{SP}, i.e., $\mathbf{Mod}_{SP}(\langle \Sigma_{SP}, \Psi_{SP} \rangle)$.

A service package specification where the only service is the service *makeAppointment* considered above, then consists of the signature Σ^{GA} and ontology Ω^{GA} as defined in Section 2, and the following specification Ψ^{GA}_{SP} for the garage appointment service. We use "String *name*" instead of only the variable "*name*" as input, which is an abbreviation for adding "*name*: String" to the precondition, and similarly for the other inputs and for the output (in which case it abbreviates part of the postcondition).

The specification says that the only appointment made through calling the service is the appointment *app* which is returned, the (week)day on which the appointment should take place is in between *from* and *to* which have been passed as parameters, and the time of day of the appointment is between 8 and 16.

$$
\begin{array}{l}
\text{makeAppointment}(\text{String } \mathit{name}, \text{WDay } \mathit{from}, \text{WDay } \mathit{to}) : \text{Appointment } \mathit{app} \\
\quad \mathsf{pre} \;\; \text{after}(\mathit{to}, \mathit{from}) \\
\quad \mathsf{post} \; \text{Appointment} \sqcap \neg(\text{Appointment@}\mathit{pre}) \equiv \{\mathit{app}\}, \\
\qquad \mathit{app} : \exists \text{hasClientName}.\{\mathit{name}\}, \\
\qquad \mathit{app} : \exists \text{hasDay}.(\exists \text{after}.\{\mathit{from}\}), \;\; \mathit{app} : \exists \text{hasDay}.(\exists \text{before}.\{\mathit{to}\}), \\
\qquad \mathit{app} : \exists \text{hasHour}.(\exists \text{after}.\{8\}), \quad\;\; \mathit{app} : \exists \text{hasHour}.(\exists \text{before}.\{16\})
\end{array}
$$

6 Matching Service Requests and Service Providers

Service package specifications can be used for specifying service providers. These service provider specifications can then be used by service requesters to determine whether a particular service provider matches their request, which can also be formulated as a service package specification. In this section, we make this matching precise by providing a model-theoretic definition of when a service request specification is matched by a service provider specification. Moreover, we provide a characterization of matching by semantic entailment over $\mathcal{SHOIN}^+$, which can be proven using standard description logic reasoners.

Our definition of matching is based on the idea that the service provider should be a *refinement* of the service request. That is, the service request specifies the behavior that the service provider is allowed to exhibit, and the specified behavior of the service provider should be within these boundaries. The idea is thus to define matching model-theoretically as inclusion of the model class of the provider specification in the model class of the request specification.

However, we cannot define this model class inclusion directly in this way, since we want to allow the request and the provider to be specified over different signatures. This is naturally facilitated through the use of institutions, by defining matching on the basis of a signature morphism from request to provider. In the semantic web community, techniques are being developed for aligning different ontologies [6], which could be applied in our setting for obtaining a signature morphism. Given a signature morphism from request to provider specification, we define matching as the inclusion of the reduct of the model class of the provider specification in the model class of the request specification.

Definition 10 *(matching)* Let $\langle \Sigma_{SP}^R, \Psi_{SP}^R \rangle$ and $\langle \Sigma_{SP}^P, \Psi_{SP}^P \rangle$ be service package specifications of request and provider, respectively, where $\sigma_{SP} : \Sigma_{SP}^R \to \Sigma_{SP}^P$ is an *SP* signature morphism. Then, the request is matched by the provider under σ_{SP} iff

$$\mathbf{Mod}_{SP}(\langle \Sigma_{SP}^P, \Psi_{SP}^P \rangle)|_{\sigma_{SP}} \subseteq \mathbf{Mod}_{SP}(\langle \Sigma_{SP}^R, \Psi_{SP}^R \rangle).$$

Now that we have defined matching model-theoretically, our aim is to be able to prove matching by proving particular logical relations between the ontologies and pre- and postconditions of the provider and request specifications.

The general idea is that for a particular service specification, the precondition of the provider should be weaker than the precondition of the request if the specification matches, since it should be possible to call the service at least in those cases required by the request. For the postcondition it is the other way around. The provider should at least guarantee what the request requires, i.e., the postcondition of the provider should be stronger than that of the request. These conditions are frequently used in the context of behavioral subtyping in object-oriented specification [13]. Moreover, we may assume that the provider ontology holds, because it is the provider's service which is actually executed. Also, in order to prove entailment of the request postcondition by the provider postcondition, we can assume additionally that the request precondition holds.

Intuitively, this is allowed since we can assume that the requester will guarantee that he satisfies his precondition, if he calls the service. These considerations lead to the following theorem.

Theorem 1 *(characterization of matching by semantic entailment)* Let $\langle \Sigma^R_{SP}, \Psi^R_{SP} \rangle$ and $\langle \Sigma^P_{SP}, \Psi^P_{SP} \rangle$ be service package specifications of request and provider, respectively, where $\langle \Sigma^P_{SP}, \Psi^P_{SP} \rangle$ is consistent, i.e., $\mathbf{Mod}_{SP}(\langle \Sigma^P_{SP}, \Psi^P_{SP} \rangle) \neq \emptyset$, and where $\sigma_{SP} : \Sigma^R_{SP} \to \Sigma^P_{SP}$ is an SP signature morphism. Then, the request is matched by the provider under σ_{SP} according to Definition 10, if the following holds.

Let $\Sigma^R_{SP} = (\langle \Sigma^R, \Omega^R \rangle, Servs^R)$ and $\Sigma^P_{SP} = (\langle \Sigma^P, \Omega^P \rangle, Servs^P)$. Then for all $\langle serv^R, \mathsf{pre}^R, \mathsf{post}^R \rangle \in \Psi^R_{SP}$ two conditions hold for $\langle serv^P, \mathsf{pre}^P, \mathsf{post}^P \rangle \in \Psi^P_{SP}$, where $serv^P = \sigma_{SP}(serv^R)$, $\sigma_{\mathcal{S}^+} : \Sigma^R \to \Sigma^P$, $X_{in} = var_{in}(serv^P)$ and $X_{in,out} = var_{in}(serv^P) \cup var_{out}(serv^P)$:[2]

1. $\sigma_{\mathcal{S}^+}(\mathsf{pre}^R) \cup \Omega^P \models_{\mathcal{S}^+, \Sigma^P_{X_{in}}} \mathsf{pre}^P$
2. $\sigma_{\mathcal{S}^+}(\mathsf{pre}^R)@pre \cup \Omega^P@pre \cup \mathsf{post}^P \cup \Omega^P \models_{\mathcal{S}^+, \Sigma^{P\ bi}_{X_{in,out}}} \sigma_{\mathcal{S}^+}(\mathsf{post}^R)$

The sentences $\Omega^P@pre$ are obtained from Ω^P by adding $@pre$ to all concept names and role names, and similarly for $\sigma_{\mathcal{S}^+}(\mathsf{pre}^R)@pre$.

The proof can be found in Appendix D. Note that we do not use the request ontology Ω^R in this characterization since it is the provider's service which is actually executed. However, as mentioned above, Ω^R is plays a key role in proving a match, since a theory morphism from Ω^R to the provider ontology Ω^P is required for a signature morphism from request to provider. This theory morphism can be proven by showing that $\Omega^P \models_{\mathcal{S}^+, \Sigma} \sigma_{\mathcal{S}^+}(\Omega^R)$, where Σ is the $\mathcal{SHOIN}^+$ signature of Ω^P. Also, we require that the provider specification is consistent, since otherwise it would match with any request specification according to Definition 10, but the relation between invariants and pre- and postconditions might be such that no match can be derived according to Theorem 1.

It is also important to note that, while the pre- and postconditions are specified over the signatures $\mathcal{SHOIN}^+_{Var}$ and $\mathcal{SHOIN}^{+bi}_{Var}$, respectively, we interpret them here as $\mathcal{SHOIN}^+$ sentences over the signatures $\Sigma^P_{X_{in}}$ and $\Sigma^{P\ bi}_{X_{in,out}}$, respectively. This is possible since the sentences and semantics of $\mathcal{SHOIN}^+_{Var}$ and $\mathcal{SHOIN}^{+bi}_{Var}$ have been defined by a reduction to $\mathcal{SHOIN}^+$ over the respective $\mathcal{SHOIN}^+$ signatures. $\mathcal{SHOIN}^+(\mathbf{D})$ entailment can further be reduced to satisfiability in $\mathcal{SHOIN}(\mathbf{D})$ [11], for which a sound and complete reasoner with acceptable to very good performance exists [20].

To illustrate matching, we take the garage appointment service package specification of Section 5 as a service provider specification. We define a service request specification CA, representing a car requesting a garage appointment, as follows. The signature Σ^{CA} is defined by $N_C = \{\text{Termin}, \text{Tag}, \text{Zeichenkette}\}$, $N_R = \{\text{nach}, \text{vor}, \text{hatTag}\}$, $N_i = \{1, 2, \ldots, 24, \text{montag}, \text{dienstag}, \ldots, \text{sonntag}\}$.

[2] We use $\sigma_{\mathcal{S}^+}(\Omega)$ as a shorthand notation for $Sen_{\mathcal{S}^+}(\sigma_{\mathcal{S}^+})(\Omega)$.

These are the notions also occurring in Σ^{CA} in German. Part of the sentences of the ontology, Ω^{CA}, are the following:

$$\begin{aligned}\{\ &\exists \mathrm{hatTag}.\mathrm{Tag} \sqsubseteq \mathrm{Termin}, \mathrm{montag} : \mathrm{Tag}, \mathrm{dienstag} : \mathrm{Tag}, \ldots, \mathrm{sonntag} : \mathrm{Tag},\\ &\mathrm{nach}(\mathrm{montag}, \mathrm{montag}), \mathrm{after}(\mathrm{montag}, \mathrm{dienstag}), \ldots,\\ &\mathrm{nach}(1,1), \mathrm{nach}(1,2), \mathrm{nach}(2,2), \mathrm{nach}(1,3), \mathrm{nach}(2,3) \ldots,\\ &\mathrm{vor}(\mathrm{montag}, \mathrm{montag}), \mathrm{vor}(\mathrm{dienstag}, \mathrm{montag}), \ldots\}\end{aligned}$$

The requester is looking for a service $terminVereinbaren(name, von, bis) : ter$, specified as follows:

$$\begin{aligned}&\mathrm{terminVereinbaren}(\mathrm{Zeichenkette}\ name, \mathrm{Tag}\ von, \mathrm{Tag}\ bis) : \mathrm{Termin}\ ter\\ &\quad \mathsf{pre}\ \ \mathrm{nach}(\mathrm{dienstag}, von), \mathrm{nach}(bis, \mathrm{dienstag})\\ &\quad \mathsf{post}\ \mathrm{hatTag}(ter, \mathrm{dienstag})\end{aligned}$$

In order to determine whether the service request CA is matched by the service provider GA, we need to define a signature morphism $\sigma : \Sigma_{SP}^{\mathsf{CA}} \rightarrow \Sigma_{SP}^{\mathsf{GA}}$. Using an appropriate signature morphism from the German notions of Σ^{CA} to the corresponding English ones of Σ^{GA},[3] it can be shown that the request is matched by the service provider (see Appendix E). The request specifies a service that makes an appointment on Tuesday if *from* and *to* are set to Tuesday, but it does not matter at what time.

7 Related Work and Concluding Remarks

Regarding related work, we mention that in [2], an approach to service specification using description logic is presented that is also based on a specification of pre- and postconditions using description logic. That paper, however, considers services for which the input parameters have already been instantiated by individual names, it does not consider output of services, and it requires strong restrictions on the kind of description logic formulas used in pre- and postconditions. Moreover, it does not provide a (model-theoretic) definition of matching with accompanying characterization. Rather, it investigates several reasoning tasks that are indispensable subtasks of matching, and focuses on solving the frame problem in this context.

In this paper, we have proposed a formal specification framework for specifying the functionality of services using description logic, based on institutions. We have defined extensions of description logic and the service specification framework itself as institutions. Using this framework, we have provided a model-theoretic definition of when a service request specification is matched by a service provider specification, allowing the request and provider specification to be defined over different signatures. We have shown that matching can be characterized by a semantic entailment relation which is formulated over a particular standard description logic. Therefore, proofs of matching can be reduced to

[3] And using the complete ontologies.

standard reasoning in description logic for which one can use efficient, sound and complete description logic reasoners.

In future work, we would like to investigate adding a more abstract layer for facilitating service discovery, where not all details with respect to input and output of the service are specified. Such more abstract specifications could be used in the first phase of a two-phase approach to service discovery (see also [22]), and the approach presented in this paper would be used in the second phase. Another topic for future research is investigating an institution-independent generalization of this approach, which allows the service specification framework to be based on arbitrary institutions, rather than on description logic. Also, the integration of our approach with specifications of dynamic interaction protocols of services can be investigated.

Moreover, more extensive experimentation with the framework will have to show what kind of services are effectively specifiable using description logic. In particular, we aim to relate our approach to the well-known OWL-S [17] ontology for service specification, which is defined in the OWL language. As in this work, OWL-S views services as operations and proposes the use of pre- and postconditions for their specification. However, OWL-S does not specify how and in what language to define pre- and postconditions, it does not come with a model-theoretic interpretation of service specifications, and matching is not formally defined and characterized.

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A The Description Logic $\mathcal{SHOIN}^+$

Definition 11 *($\mathcal{SHOIN}^+$ institution)* The institution $\mathcal{SHOIN}^+ = \langle Sig_{\mathcal{S}^+}, Sen_{\mathcal{S}^+}, Mod_{\mathcal{S}^+}, \models_{\mathcal{S}^+,\Sigma}\rangle$ is defined as follows:

- $Sig_{\mathcal{S}^+}$ is the category of $\mathcal{SHOIN}^+$ signatures.
  - A $\mathcal{SHOIN}^+$ signature Σ is a tuple $\langle N_C, N_R, N_i\rangle$, where N_C with typical element A is a set of concept names, $N_R = R \cup R^-$, where R is a set of (basic) role names and $R^- = \{r^- \mid r \in R\}$, is a set of role names with typical element r, and N_i with typical element a is a set of individual names. The sets N_C, N_R, and N_i are pairwise disjoint.
  - Let $\Sigma = \langle N_C, N_R, N_i\rangle$ and $\Sigma' = \langle N'_C, N'_R, N'_i\rangle$ be $\mathcal{SHOIN}^+$ signatures. A $\mathcal{SHOIN}^+$ signature morphism $\sigma_{\mathcal{S}^+} : \Sigma \to \Sigma'$ consists of a mapping of the concept names of N_C to concept names of N'_C, a mapping of the role names of N_R to role names of N'_R, and a mapping of the individual names of N_i to individual names of N'_i.
- The functor $Sen_{\mathcal{S}^+} : Sig_{\mathcal{S}^+} \to Set$ maps
  - each $\mathcal{SHOIN}^+$ signature $\Sigma = \langle N_C, N_R, N_i\rangle$ to the set of $\mathcal{SHOIN}^+$ sentences, as specified in Definition 12.
  - each $\mathcal{SHOIN}^+$ signature morphism $\sigma_{\mathcal{S}^+} : \Sigma \to \Sigma'$ to the obvious translation function $Sen_{\mathcal{S}^+}(\sigma_{\mathcal{S}^+})$ which transforms Σ-sentences to Σ'-sentences.
- The functor $Mod_{\mathcal{S}^+} : (Sig_{\mathcal{S}^+})^{op} \to Cat$ maps
  - each $\mathcal{SHOIN}^+$ signature $\Sigma = \langle N_C, N_R, N_i\rangle$ to the category of $\mathcal{SHOIN}^+$ models as specified in Definition 13, and Σ-homomorphisms for each $\Sigma \in |Sig_{\mathcal{S}^+}|$.
  - each $\mathcal{SHOIN}^+$ signature morphism $\sigma_{\mathcal{S}^+} : \Sigma \to \Sigma'$ to the reduct functor $Mod_{\mathcal{S}^+}(\sigma_{\mathcal{S}^+}) : Mod_{\mathcal{S}^+}(\Sigma') \to Mod_{\mathcal{S}^+}(\Sigma)$, as specified in Definition 13.
- For each $\mathcal{SHOIN}^+$ signature $\Sigma \in |Sig_{\mathcal{S}^+}|$, the satisfaction relation $\models_{\mathcal{S}^+,\Sigma} \subseteq Mod_{\mathcal{S}^+}(\Sigma) \times Sen_{\mathcal{S}^+}(\Sigma)$ is as specified in Definition 14.

This definition of $\mathcal{SHOIN}^+$ as an institution is similar to the way OWL DL, the semantic web markup language corresponding to $\mathcal{SHOIN}(\mathbf{D})$, was defined as an institution in [14, 15]. The proof of the satisfaction condition can be found in [14].

Definition 12 *($\mathcal{SHOIN}^+$ sentences)* Let $\Sigma = \langle N_C, N_R, N_i\rangle \in |Sig_{\mathcal{S}^+}|$ be a $\mathcal{SHOIN}^+$ signature, and let $A \in N_C$, $r \in N_R$, and $a, a_1, a_2 \in N_i$. The sentences $Sen_{\mathcal{S}^+}(\Sigma)$ are then the axioms ϕ as defined below.

$$\begin{aligned} C &::= A \mid \top \mid \bot \mid \neg C \mid C_1 \sqcap C_2 \mid C_1 \sqcup C_2 \mid \{a\} \mid \exists r.C \mid \forall r.C \mid \geq n\ r \mid \leq n\ r \\ \phi &::= C_1 \sqsubseteq C_2 \mid r_1 \sqsubseteq r_2 \mid a : C \mid r(a_1, a_2) \mid \exists C \mid \mathrm{Trans}(r) \mid a_1 = a_2 \mid a_1 \neq a_2 \end{aligned}$$

Definition 13 *($\mathcal{SHOIN}^+$ models)* Let $\Sigma = \langle N_C, N_R, N_i\rangle \in |Sig_{\mathcal{S}^+}|$ be a $\mathcal{SHOIN}^+$ signature, where $N_R = R \cup R^-$ as specified in Definition 11. A model or interpretation $\mathcal{I}$ for $\mathcal{SHOIN}^+$ consists of a non-empty domain $\Delta^{\mathcal{I}}$ of individuals and an interpretation function $\cdot^{\mathcal{I}}$ which maps each concept name $A \in N_C$

to a subset $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$ of the domain, each basic role name $r \in R$ to a binary relation $r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ on the domain, and each individual name $a \in N_i$ to an element $a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$. The interpretation of an inverse role $r^- \in R^-$ is defined as follows: $(r^-)^{\mathcal{I}} = \{(y,x) \mid (x,y) \in r^{\mathcal{I}}\}$.

The σ-reduct $Mod_{\mathcal{S}^+}(\sigma_{\mathcal{S}^+}) : Mod_{\mathcal{S}^+}(\Sigma') \to Mod_{\mathcal{S}^+}(\Sigma)$ for $\mathcal{SHOIN}^+$ signature morphism $\sigma_{\mathcal{S}^+} : \Sigma \to \Sigma'$ is denoted by $-|_{\sigma_{\mathcal{S}^+}}$ and defined as follows. Let $\mathcal{I}' = (\Delta^{\mathcal{I}'}, \cdot^{\mathcal{I}'}) \in |Mod_{\mathcal{S}^+}(\Sigma')|$ be a Σ'-model. The reduct of $\mathcal{I}'$ is defined in two steps. The reduct of $\Delta^{\mathcal{I}'}$, denoted by $\Delta^{\mathcal{I}'}|_{\sigma_{\mathcal{S}^+}}$, is defined as $\Delta^{\mathcal{I}'}$. The reduct of $\cdot^{\mathcal{I}'}$, denoted by $\cdot^{\mathcal{I}'}|_{\sigma_{\mathcal{S}^+}}$, is defined as follows, where $A \in N_C$, $r \in N_R$, and $a \in N_i$.

$$\begin{aligned}
A^{\mathcal{I}'|_{\sigma_{\mathcal{S}^+}}} &= \sigma(A)^{\mathcal{I}'} \\
r^{\mathcal{I}'|_{\sigma_{\mathcal{S}^+}}} &= \sigma(r)^{\mathcal{I}'} \\
a^{\mathcal{I}'|_{\sigma_{\mathcal{S}^+}}} &= \sigma(a)^{\mathcal{I}'}
\end{aligned}$$

Definition 14 *($\mathcal{SHOIN}^+$ satisfaction relation)* Let $\Sigma \in |Sig_{\mathcal{S}^+}|$ be a $\mathcal{SHOIN}^+$ signature and let $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}}) \in |Mod_{\mathcal{S}^+}(\Sigma)|$ be a Σ-model. The satisfaction relation $\models_{\mathcal{S}^+,\Sigma}$ is defined by first lifting the interpretation function $\cdot^{\mathcal{I}}$ to composed concepts as follows.

$$\begin{aligned}
\top^{\mathcal{I}} &= \Delta^{\mathcal{I}} \\
\bot^{\mathcal{I}} &= \emptyset \\
(\neg C)^{\mathcal{I}} &= \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}} \\
(C_1 \sqcap C_2)^{\mathcal{I}} &= C_1^{\mathcal{I}} \cap C_2^{\mathcal{I}} \\
(C_1 \sqcup C_2)^{\mathcal{I}} &= C_1^{\mathcal{I}} \cup C_2^{\mathcal{I}} \\
\{a\}^{\mathcal{I}} &= \{a^{\mathcal{I}}\} \\
\exists r.C^{\mathcal{I}} &= \{x \in \Delta^{\mathcal{I}} \mid \exists y : (x,y) \in r^{\mathcal{I}} \text{ and } y \in C^{\mathcal{I}}\} \\
\forall r.C^{\mathcal{I}} &= \{x \in \Delta^{\mathcal{I}} \mid \forall y : (x,y) \in r^{\mathcal{I}} \Rightarrow y \in C^{\mathcal{I}}\} \\
\geq n\ r^{\mathcal{I}} &= \{x \in \Delta^{\mathcal{I}} \mid \sharp(\{y \in \Delta^{\mathcal{I}} \mid (x,y) \in r^{\mathcal{I}}\}) \geq n\} \\
\leq n\ r^{\mathcal{I}} &= \{x \in \Delta^{\mathcal{I}} \mid \sharp(\{y \in \Delta^{\mathcal{I}} \mid (x,y) \in r^{\mathcal{I}}\}) \leq n\}
\end{aligned}$$

We can now define the satisfaction of an axiom by an interpretation $\mathcal{I}$.

$$\begin{aligned}
\mathcal{I} \models_{\mathcal{S}^+,\Sigma} C_1 \sqsubseteq C_2 &\Leftrightarrow C_1^{\mathcal{I}} \subseteq C_2^{\mathcal{I}} \\
\mathcal{I} \models_{\mathcal{S}^+,\Sigma} a : C &\Leftrightarrow a^{\mathcal{I}} \in C^{\mathcal{I}} \\
\mathcal{I} \models_{\mathcal{S}^+,\Sigma} r(a_1, a_2) &\Leftrightarrow (a_1^{\mathcal{I}}, a_2^{\mathcal{I}}) \in r^{\mathcal{I}} \\
\mathcal{I} \models_{\mathcal{S}^+,\Sigma} \exists C &\Leftrightarrow \exists i : i \in C^{\mathcal{I}} \\
\mathcal{I} \models_{\mathcal{S}^+,\Sigma} \mathrm{Trans}(r) &\Leftrightarrow r^{\mathcal{I}} \text{ is transitive} \\
\mathcal{I} \models_{\mathcal{S}^+,\Sigma} a_1 = a_2 &\Leftrightarrow a_1^{\mathcal{I}} = a_2^{\mathcal{I}} \\
\mathcal{I} \models_{\mathcal{S}^+,\Sigma} a_1 \neq a_2 &\Leftrightarrow a_1^{\mathcal{I}} \neq a_2^{\mathcal{I}}
\end{aligned}$$

The satisfaction relation $\models_{\mathcal{S}^+,\Sigma}$ is lifted to sets of axioms in the usual way.

Definition 15 *($\mathcal{SHOIN}^+$ ontology)* We define a $\mathcal{SHOIN}^+$ ontology as a presentation $\langle \Sigma, \Omega \rangle$, where $\Sigma \in |Sig_{\mathcal{S}^+}|$ and $\Omega \subseteq Sen_{\mathcal{S}^+}(\Sigma)$. Its semantics is the class of Σ-models satisfying the axioms in Ω, i.e., $\mathbf{Mod}_{\mathcal{S}^+}(\langle \Sigma, \Omega \rangle)$.

B Extensions of $\mathcal{SHOIN}^+$

Definition 16 *($\mathcal{SHOIN}^+_{Var}$ institution)* The institution $\mathcal{SHOIN}^+_{Var} = \langle Sig_{\mathcal{S}^+_{Var}}, Sen_{\mathcal{S}^+_{Var}}, Mod_{\mathcal{S}^+_{Var}}, \models_{\mathcal{S}^+_{Var},\Sigma_X}\rangle$ is defined as follows:

- $Sig_{\mathcal{S}^+_{Var}}$ is the category of $\mathcal{SHOIN}^+_{Var}$ signatures.
  - A $\mathcal{SHOIN}^+_{Var}$ signature is a pair $\langle \Sigma, X\rangle$ where $\Sigma = \langle N_C, N_R, N_i\rangle \in |Sig_{\mathcal{S}^+}|$ and X is a set of variables such that $X \cap N_i = \emptyset$.
  - Let $\langle \Sigma, X\rangle$ and $\langle \Sigma', X'\rangle$ be $\mathcal{SHOIN}^+_{Var}$ signatures. A $\mathcal{SHOIN}^+_{Var}$ signature morphism $\sigma_{\mathcal{S}^+_{Var}} : \langle \Sigma, X\rangle \to \langle \Sigma', X'\rangle$ consists of the signature morphism $\sigma_{\mathcal{S}^+} : \Sigma \to \Sigma'$, and a mapping of the variables of X to variables of X'.
- The functor $Sen_{\mathcal{S}^+_{Var}} : Sig_{\mathcal{S}^+_{Var}} \to Set$ maps
  - each $\mathcal{SHOIN}^+_{Var}$ signature $\langle \Sigma, X\rangle$ to the set of $\mathcal{SHOIN}^+_{Var}$ sentences as follows. Let $\Sigma = \langle N_C, N_R, N_i\rangle$ and let $\Sigma_X = \langle N_C, N_R, N_i \cup X\rangle$. We now define $Sen_{\mathcal{S}^+_{Var}}(\langle \Sigma, X\rangle) \triangleq Sen_{\mathcal{S}^+}(\Sigma_X)$.
  - each $\mathcal{SHOIN}^+_{Var}$ signature morphism $\sigma_{\mathcal{S}^+_{Var}} : \langle \Sigma, X\rangle \to \langle \Sigma', X'\rangle$ to the obvious translation function $Sen_{\mathcal{S}^+_{Var}}(\sigma_{\mathcal{S}^+_{Var}})$ which transforms $\langle \Sigma, X\rangle$-sentences to $\langle \Sigma', X'\rangle$-sentences.
- The functor $Mod_{\mathcal{S}^+_{Var}} : (Sig_{\mathcal{S}^+_{Var}})^{op} \to Cat$ maps
  - each $\mathcal{SHOIN}^+_{Var}$ signature $\langle \Sigma, X\rangle$ to the category of $\mathcal{SHOIN}^+_{Var}$ models. A $\mathcal{SHOIN}^+_{Var}$ model is a pair $(\mathcal{I}, \rho)$ where $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}}) \in |Mod_{\mathcal{S}^+}(\Sigma)|$ and $\rho : X \to \Delta^{\mathcal{I}}$ is a valuation, and the morphisms are the $\langle \Sigma, X\rangle$-homomorphisms for each $\langle \Sigma, X\rangle \in |Sig_{\mathcal{S}^+_{Var}}|$.
  - each $\mathcal{SHOIN}^+_{Var}$ signature morphism $\sigma_{\mathcal{S}^+_{Var}} : \langle \Sigma, X\rangle \to \langle \Sigma', X'\rangle$ to the reduct functor $Mod_{\mathcal{S}^+_{Var}}(\sigma_{\mathcal{S}^+_{Var}}) : Mod_{\mathcal{S}^+_{Var}}(\langle \Sigma', X'\rangle) \to Mod_{\mathcal{S}^+_{Var}}(\langle \Sigma, X\rangle)$. The reduct $(\mathcal{I}', \rho')|_{\sigma_{\mathcal{S}^+_{Var}}}$ consists of $\mathcal{I}'|_{\sigma_{\mathcal{S}^+}}$ and $\rho'|_{\sigma_{\mathcal{S}^+_{Var}}}$, where $\rho'|_{\sigma_{\mathcal{S}^+_{Var}}}(x) = \rho'(\sigma_{\mathcal{S}^+_{Var}}(x))$ with $x \in X$.
- For each $\mathcal{SHOIN}^+_{Var}$ signature $\langle \Sigma, X\rangle \in |Sig_{\mathcal{S}^+_{Var}}|$, the satisfaction relation $\models_{\mathcal{S}^+_{Var},\langle \Sigma, X\rangle} \subseteq Mod_{\mathcal{S}^+_{Var}}(\langle \Sigma, X\rangle) \times Sen_{\mathcal{S}^+_{Var}}(\langle \Sigma, X\rangle)$ is defined as follows by means of a reduction to $\models_{\mathcal{S}^+,\Sigma_X}$. Let $(\mathcal{I}, \rho) \in Mod_{\mathcal{S}^+_{Var}}(\langle \Sigma, X\rangle)$ and let $\mathcal{I}_\rho \in Mod_{\mathcal{S}^+}(\Sigma_X)$ where $\Delta^{\mathcal{I}_\rho} \triangleq \Delta^{\mathcal{I}}$, and $\cdot^{\mathcal{I}_\rho} \triangleq \cdot^{\mathcal{I}}$ for concept names, role names, and individual names of Σ, and $\cdot^{\mathcal{I}_\rho}(x) \triangleq \rho(x)$ for $x \in X$.
  We now define $(\mathcal{I}, \rho) \models_{\mathcal{S}^+_{Var},\langle \Sigma, X\rangle} \phi \triangleq \mathcal{I}_\rho \models_{\mathcal{S}^+,\Sigma_X} \phi$ for $\phi \in Sen_{\mathcal{S}^+_{Var}}(\langle \Sigma, X\rangle)$ and thus by definition also $\phi \in Sen_{\mathcal{S}^+}(\Sigma_X)$.

Proposition 1 *(satisfaction condition of $\mathcal{SHOIN}^+_{Var}$)* The satisfaction condition of $\mathcal{SHOIN}^+_{Var}$ holds.

Proof: The satisfaction condition holds, since $\mathcal{SHOIN}^+_{Var}$ signatures and models can be reduced to $\mathcal{SHOIN}^+$ signatures and models, where $\mathcal{SHOIN}^+_{Var}$ sentences over the $\mathcal{SHOIN}^+_{Var}$ signature are interpreted as $\mathcal{SHOIN}^+$ sentences over the corresponding $\mathcal{SHOIN}^+$ signature.

To be more specific, let $\langle \Sigma, X\rangle$ and $\langle \Sigma', X'\rangle$ be $\mathcal{SHOIN}^{+}_{Var}$ signatures with $\sigma_{\mathcal{S}^{+}_{Var}} : \langle \Sigma, X\rangle \rightarrow \langle \Sigma', X'\rangle$ a signature morphism, let $\phi \in Sen_{\mathcal{S}^{+}_{Var}}(\langle \Sigma, X\rangle)$, and let $(\mathcal{I}', \rho') \in |Mod_{\mathcal{S}^{+}_{Var}}(\langle \Sigma', X'\rangle)|$. Then, we have $(\mathcal{I}', \rho') \models_{\mathcal{S}^{+}_{Var}, \langle \Sigma', X'\rangle} \sigma(\phi) \Leftrightarrow (\mathcal{I}', \rho')|_{\sigma_{\mathcal{S}^{+}_{Var}}} \models_{\mathcal{S}^{+}_{Var}, \langle \Sigma, X\rangle} \phi$ iff $\mathcal{I}'_{\rho'} \models_{\mathcal{S}^{+}, \Sigma'_{X'}} \sigma(\phi) \Leftrightarrow \mathcal{I}'_{\rho'}|_{\sigma_{\mathcal{S}^{+}}} \models_{\mathcal{S}^{+}, \Sigma_X} \phi$, by Definition 16. $\square$

Definition 17 *($\mathcal{SHOIN}^{+bi}_{Var}$ institution)* The institution $\mathcal{SHOIN}^{+bi}_{Var} = \langle Sig_{\mathcal{S}^{+bi}_{Var}}, Sen_{\mathcal{S}^{+bi}_{Var}}, Mod_{\mathcal{S}^{+bi}_{Var}}, \models_{\mathcal{S}^{+bi}_{Var}, \Sigma}\rangle$ is defined as follows:

- The $\mathcal{SHOIN}^{+bi}_{Var}$ signatures are the $\mathcal{SHOIN}^{+}_{Var}$ signatures, i.e., $Sig_{\mathcal{S}^{+bi}_{Var}} = Sig_{\mathcal{S}^{+}_{Var}}$, with corresponding morphisms.
- The functor $Sen_{\mathcal{S}^{+bi}_{Var}} : Sig_{\mathcal{S}^{+bi}_{Var}} \rightarrow Set$ maps
  - each $\mathcal{SHOIN}^{+bi}_{Var}$ signature $\langle \Sigma, X\rangle$ to the set of $\mathcal{SHOIN}^{+bi}_{Var}$ sentences as follows. Let $\Sigma = \langle N_C, N_R, N_i\rangle$ and let $\Sigma^{bi} = \langle N_C \cup N_C^{bi}, N_R \cup N_R^{bi}, N_i\rangle$ where $N_C^{bi} = \{A@pre \mid A \in N_C\}$ and $N_R^{bi} = \{r@pre \mid r \in N_R\}$. We define $Sen_{\mathcal{S}^{+bi}_{Var}}(\langle \Sigma, X\rangle) \triangleq Sen_{\mathcal{S}^{+}_{Var}}(\langle \Sigma^{bi}, X\rangle) = Sen_{\mathcal{S}^{+}}(\Sigma^{bi}_X)$.
  - each $\mathcal{SHOIN}^{+bi}_{Var}$ signature morphism $\sigma_{\mathcal{S}^{+bi}_{Var}} : \langle \Sigma, X\rangle \rightarrow \langle \Sigma', X'\rangle$ to the obvious translation function $Sen_{\mathcal{S}^{+bi}_{Var}}(\sigma_{\mathcal{S}^{+bi}_{Var}})$.
- The functor $Mod_{\mathcal{SHOIN}^{+bi}_{Var}} : (Sig_{\mathcal{SHOIN}^{+bi}_{Var}})^{op} \rightarrow Cat$ maps
  - each $\mathcal{SHOIN}^{+bi}_{Var}$ signature $\langle \Sigma, X\rangle$ to the category of $\mathcal{SHOIN}^{+bi}_{Var}$ models. A $\mathcal{SHOIN}^{+bi}_{Var}$ model is a triple $(\mathcal{I}_1, \mathcal{I}_2, \rho)$ where $\mathcal{I}_1, \mathcal{I}_2 \in |Mod_{\mathcal{S}^{+}}(\Sigma)|$, $\mathcal{I}_1 = (\Delta^{\mathcal{I}_1}, \cdot^{\mathcal{I}_1})$, $\mathcal{I}_2 = (\Delta^{\mathcal{I}_2}, \cdot^{\mathcal{I}_2})$, $\Delta^{\mathcal{I}_1} = \Delta^{\mathcal{I}_2}$, and $a^{\mathcal{I}_1} = a^{\mathcal{I}_2}$ for all $a \in N_i$, and $\rho : X \rightarrow \Delta$ is a valuation where $\Delta \triangleq \Delta^{\mathcal{I}_1} (= \Delta^{\mathcal{I}_2})$. The morphisms are the $\langle \Sigma, X\rangle$-homomorphisms for each $\langle \Sigma, X\rangle \in |Sig_{\mathcal{S}^{+bi}_{Var}}|$.
  - each $\mathcal{SHOIN}^{+bi}_{Var}$ signature morphism $\sigma_{\mathcal{S}^{+bi}_{Var}} : \langle \Sigma, X\rangle \rightarrow \langle \Sigma', X'\rangle$ to the reduct functor $Mod_{\mathcal{S}^{+bi}_{Var}}(\sigma_{\mathcal{S}^{+bi}_{Var}}) : Mod_{\mathcal{S}^{+bi}_{Var}}(\langle \Sigma', X'\rangle) \rightarrow Mod_{\mathcal{S}^{+bi}_{Var}}(\langle \Sigma, X\rangle)$. The reduct $(\mathcal{I}'_1, \mathcal{I}'_2, \rho')|_{\sigma_{\mathcal{S}^{+bi}_{Var}}}$ consists of $\mathcal{I}'_1|_{\sigma_{\mathcal{S}^{+}}}$, $\mathcal{I}'_2|_{\sigma_{\mathcal{S}^{+}}}$, and $\rho'|_{\sigma_{\mathcal{S}^{+}_{Var}}}$.
- For each $\mathcal{SHOIN}^{+bi}_{Var}$ signature $\langle \Sigma, X\rangle \in |Sig_{\mathcal{S}^{+bi}_{Var}}|$, the satisfaction relation $\models_{\mathcal{S}^{+bi}_{Var}, \langle \Sigma, X\rangle} \subseteq Mod_{\mathcal{S}^{+bi}_{Var}}(\langle \Sigma, X\rangle) \times Sen_{\mathcal{S}^{+bi}_{Var}}(\langle \Sigma, X\rangle)$ is defined as follows by means of a reduction to $\models_{\mathcal{S}^{+}, \Sigma^{bi}_X}$.

  Let $(\mathcal{I}_1, \mathcal{I}_2, \rho) \in Mod_{\mathcal{S}^{+bi}_{Var}}(\langle \Sigma, X\rangle)$ and let $\overset{\curlywedge}{\mathcal{I}}_\rho \in Mod_{\mathcal{S}^{+}}(\Sigma^{bi}_X)$ be defined as follows: $\Delta^{\overset{\curlywedge}{\mathcal{I}}_\rho} = \Delta^{\mathcal{I}_1} (= \Delta^{\mathcal{I}_2})$, $\cdot^{\overset{\curlywedge}{\mathcal{I}}_\rho} = \cdot^{(\mathcal{I}_2)_\rho}$ for concept names A, role names r, and individual names a of Σ, and $\cdot^{\overset{\curlywedge}{\mathcal{I}}_\rho} = \cdot^{(\mathcal{I}_1)_\rho}$ for concept names $A@pre$ and role names $r@pre$, where $(\mathcal{I}_1)_\rho$ and $(\mathcal{I}_2)_\rho$ are the extension of $\mathcal{I}_1$ and $\mathcal{I}_2$, respectively, to variables as defined above.

  We now define $(\mathcal{I}_1, \mathcal{I}_2, \rho) \models_{\mathcal{S}^{+bi}_{Var}, \langle \Sigma, X\rangle} \phi \triangleq \overset{\curlywedge}{\mathcal{I}}_\rho \models_{\mathcal{S}^{+}, \Sigma^{bi}_X} \phi$ for $\phi \in Sen_{\mathcal{S}^{+bi}_{Var}}(\langle \Sigma, X\rangle)$ and thus by definition also $\phi \in Sen_{\mathcal{S}^{+}}(\Sigma^{bi}_X)$.

Proposition 2 *(satisfaction condition of $\mathcal{SHOIN}^{+bi}_{Var}$)* The satisfaction condition of $\mathcal{SHOIN}^{+bi}_{Var}$ holds.

Proof: The proof is by a reduction to $\mathcal{SHOIN}^+$, similar to the proof of Proposition 1. $\square$

C Service Specification using Description Logic

Definition 18 *(service)* A service $serv = servName([X_{in}]) : [X_{out}]$ consists of a service name $servName$, and sequences of input and output variables $[X_{in}]$ and $[X_{out}]$, respectively, such that all x in $[X_{in}]$ and $[X_{out}]$ are distinct. We use $var_{in}(serv)$ and $var_{out}(serv)$ to denote the sets of input and output variables of $serv$, respectively.

Definition 19 *(service package (SP) institution)* The institution $SP = \langle Sig_{SP}, Sen_{SP}, Mod_{SP}, \models_{SP,(\langle\Sigma,\Omega\rangle,Servs)}\rangle$ is defined as follows:

- Sig_{SP} is the category of SP signatures.
  - An SP signature is a pair $(\langle\Sigma,\Omega\rangle, Servs)$ where $\langle\Sigma,\Omega\rangle$ is a $\mathcal{SHOIN}^+$ ontology (see Definition 15), and $Servs$ is a set of services.
  - Let $(\langle\Sigma,\Omega\rangle, Servs)$ and $(\langle\Sigma',\Omega'\rangle, Servs')$ be SP signatures. An SP signature morphism $\sigma_{SP} : (\langle\Sigma,\Omega\rangle, Servs) \to (\langle\Sigma',\Omega'\rangle, Servs')$ consists of a theory morphism $\sigma_{Ont} : \langle\Sigma, Cl_\Sigma(\Omega)\rangle \to \langle\Sigma', Cl_{\Sigma'}(\Omega')\rangle$, and a mapping of each service $serv \in Servs$ to a service $serv' \in Servs'$, such that for each mapping from $serv$ to $serv'$ it holds that $serv$ and $serv'$ have the same number of input variables and the same number of output variables.
- The functor $Sen_{SP} : Sig_{SP} \to Set$ maps
  - each SP signature $(\langle\Sigma,\Omega\rangle, Servs)$ to the set of SP sentences as follows. An SP sentence is a triple $\langle serv, \mathsf{pre}, \mathsf{post}\rangle$, where $serv$ is a service, and

    $$\mathsf{pre} \subseteq Sen_{\mathcal{S}^+_{Var}}(\langle\Sigma, X_{in}\rangle),$$
    $$\mathsf{post} \subseteq Sen_{\mathcal{SHOIN}^{+bi}_{Var}}(\langle\Sigma, X_{in,out}\rangle),$$

    where here and in the following $X_{in} = var_{in}(serv)$, $X_{out} = var_{out}(serv)$, and $X_{in,out} = var_{in}(serv) \cup var_{out}(serv)$.
  - each SP signature morphism $\sigma_{SP} : (\langle\Sigma,\Omega\rangle, Servs) \to (\langle\Sigma',\Omega'\rangle, Servs')$ to the obvious translation function $Sen_{SP}(\sigma_{SP})$.
- The functor $Mod_{SP} : (Sig_{SP})^{op} \to Cat$ maps
  - each SP signature $(\langle\Sigma,\Omega\rangle, Servs)$ to the category of SP models. An SP model for this signature is a non-deterministic total labeled transition system with outputs $\mathcal{T} = (Q, \delta)$, where $Q \subseteq \mathbf{Mod}_{\mathcal{S}^+}(\langle\Sigma,\Omega\rangle)$ is a set of states and δ is a set of transitions between states, defined as follows. Let $Label = \{(serv, \rho_{in}) \mid serv \in Servs, \rho_{in} : X_{in} \to \Delta\}$, where $\Delta = \bigcup\{\Delta^{\mathcal{I}} \mid \mathcal{I} \in Q\}$ and let $Output$ be the set of valuations $\rho_{out} : X \to \Delta$ where X is an arbitrary set of variables. Then $\delta \subseteq Q \times Label \times (Q \times Output)$ such that for all $(\mathcal{I}, (serv, \rho_{in}), (\mathcal{I}', \rho_{out})) \in \delta$ we have $\rho_{in} : X_{in} \to \Delta^{\mathcal{I}}$ and $\rho_{out} : X_{out} \to \Delta^{\mathcal{I}'}$, and $\mathcal{T}$ is total, i.e., for all $\mathcal{I} \in Q$ it holds that for all $l \in Label$ there is an $\mathcal{I}', \rho_{out}$ such that $(\mathcal{I}, l, (\mathcal{I}', \rho_{out})) \in \delta$.

A morphism $\mathcal{T} \to \mathcal{T}'$, where $\mathcal{T} = (Q_{\mathcal{T}}, \delta_{\mathcal{T}})$ and $\mathcal{T}' = (Q_{\mathcal{T}'}, \delta_{\mathcal{T}'})$, consists of a function $f : Q_{\mathcal{T}} \to Q_{\mathcal{T}'}$ such that if $f(\mathcal{I}) = \mathcal{I}'$ then there is a Σ-morphism $\mathcal{I} \to \mathcal{I}'$, and a mapping of each $(\mathcal{I}_1, (serv, \rho_{in}), (\mathcal{I}_2, \rho_{out})) \in \delta_{\mathcal{T}}$ to a transition $(f(\mathcal{I}_1), (serv, g(\rho_{in})), (f(\mathcal{I}_2), g(\rho_{out}))) \in \delta_{\mathcal{T}'}$ where $g(\rho_{in}) : X \to \Delta_{f(\mathcal{I}_1)}$ (with $\rho_{in} : X \to \Delta_{\mathcal{I}_1}$) is a valuation such that $g(\rho_{in})(x) = d(a)$ if $\rho_{in}(x) = a$, where $d : \Delta_{\mathcal{I}_1} \to \Delta_{f(\mathcal{I}_1)}$ is the domain mapping function of the Σ-morphism $\mathcal{I}_1 \to f(\mathcal{I}_1)$, and similarly for ρ_{out}.

- each SP signature morphism $\sigma_{SP} : (\langle \Sigma, \Omega \rangle, Servs) \to (\langle \Sigma', \Omega' \rangle, Servs')$ to the reduct functor $Mod_{SP}(\sigma_{SP}) : Mod_{SP}((\langle \Sigma', \Omega' \rangle, Servs')) \to Mod_{SP}((\langle \Sigma, \Omega \rangle, Servs))$. The reduct $\mathcal{T}'|_{\sigma_{SP}}$ where $\mathcal{T}' = (Q', \delta')$ is $(Q'|_{\sigma_{Ont}}, \delta'|_{\sigma_{SP}})$, where $Q'|_{\sigma_{Ont}} = \{\mathcal{I}'|_{\sigma_{Ont}} \mid \mathcal{I}' \in Q'\}$, and $\delta'|_{\sigma_{SP}}$ are all transitions $(\mathcal{I}_1|_{\sigma_{Ont}}, (serv, \rho_{in}|_{\sigma_{\mathcal{S}^+_{Var}}}), \mathcal{I}_2|_{\sigma_{Ont}}, \rho_{out}|_{\sigma_{\mathcal{S}^+_{Var}}})$ such that $(\mathcal{I}_1, (\sigma_{SP}(serv), \rho_{in}), \mathcal{I}_2, \rho_{out}) \in \delta'$. Note that $Q'|_{\sigma_{Ont}} \subseteq \mathbf{Mod}_{\mathcal{S}^+}(\langle \Sigma, \Omega \rangle)$, since σ_{Ont} is a theory morphism (see above).

– For each SP signature $(\langle \Sigma, \Omega \rangle, Servs)$, the satisfaction relation $\models_{SP,(\langle \Sigma, \Omega \rangle, Servs)} \subseteq Mod_{SP}((\langle \Sigma, \Omega \rangle, Servs)) \times Sen_{SP}((\langle \Sigma, \Omega \rangle, Servs))$ is defined as follows. Let $\Sigma_{SP} = (\langle \Sigma, \Omega \rangle, Servs)$ be an SP signature, and let $\mathcal{T} = (Q, \delta) \in Mod_{SP}((\langle \Sigma, \Omega \rangle, Servs))$. We define $\mathcal{T} \models_{SP, \Sigma_{SP}} \langle serv, \mathsf{pre}, \mathsf{post} \rangle$ iff for all $\mathcal{I} \in Q$ and for all $\rho_{in} : X_{in} \to \Delta^{\mathcal{I}}$, if $(\mathcal{I}, \rho_{in}) \models_{\mathcal{S}^+_{Var}, \langle \Sigma, X_{in} \rangle} \mathsf{pre}$ then for all $(\mathcal{I}, (serv, \rho_{in}), \mathcal{I}', \rho_{out}) \in \delta$ it holds that $(\mathcal{I}, \mathcal{I}', \rho_{in,out}) \models_{\mathcal{S}^{+bi}_{Var}, \langle \Sigma, X_{in,out} \rangle}$ post. We use $\rho_{in,out}$ to denote the merging of the two valuations ρ_{in} and ρ_{out} to one valuation in the obvious way.

Proposition 3 *(satisfaction condition of SP)* The satisfaction condition of SP holds.

Proof: Let $(\langle \Sigma, \Omega \rangle, Servs)$ and $(\langle \Sigma', \Omega' \rangle, Servs')$ be SP signatures with $\sigma_{SP} : (\langle \Sigma, \Omega \rangle, Servs) \to (\langle \Sigma', \Omega' \rangle, Servs')$ a signature morphism, let $\langle serv, \mathsf{pre}, \mathsf{post} \rangle \in Sen_{SP}((\langle \Sigma, \Omega \rangle, Servs))$, and let $\mathcal{T}' = (Q', \delta') \in |Mod_{SP}((\langle \Sigma', \Omega' \rangle, Servs'))|$ and $\mathcal{T}'|_{\sigma_{SP}} = (Q, \delta)$. Let $X_{in} = var_{in}(serv)$, $X_{out} = var_{out}(serv)$, and $X_{in,out} = var_{in}(serv) \cup var_{out}(serv)$ To prove:

$$\mathcal{T}'|_{\sigma_{SP}} \models_{SP,(\langle \Sigma, \Omega \rangle, Servs)} \langle serv, \mathsf{pre}, \mathsf{post} \rangle \Leftrightarrow \mathcal{T}' \models_{SP,(\langle \Sigma', \Omega' \rangle, Servs')} \sigma_{Ont}(\langle serv, \mathsf{pre}, \mathsf{post} \rangle)$$

$(\Leftarrow)$ Assume $\mathcal{T}' \models_{SP,(\langle \Sigma', \Omega' \rangle, Servs')} \sigma_{Ont}(\langle serv, \mathsf{pre}, \mathsf{post} \rangle)$ $(*)$. Let $\mathcal{I}'_1 \in Q'$, let $\rho'_{in} : X'_{in} \to \Delta^{\mathcal{I}'_1}$, and let $(\mathcal{I}'_1, \rho'_{in}) \models_{\mathcal{S}^+_{Var}, \langle \Sigma', X'_{in} \rangle} \sigma_{\mathcal{S}^+_{Var}}(\mathsf{pre})$. Then, by Definition 19 and $(*)$, we have for all $(\mathcal{I}'_1, (serv, \rho'_{in}), \mathcal{I}'_2, \rho'_{out}) \in \delta'$: $(\mathcal{I}'_1, \mathcal{I}'_2, \rho'_{in,out}) \models_{\mathcal{S}^{+bi}_{Var}, \langle \Sigma', X'_{in,out} \rangle} \sigma_{\mathcal{S}^{+bi}_{Var}}(\mathsf{post})$. We have $(\mathcal{I}'_1, \rho'_{in})|_{\sigma_{\mathcal{S}^+_{Var}}} \models_{\mathcal{S}^+_{Var}, \langle \Sigma, X_{in} \rangle} \mathsf{pre}$, using the satisfaction condition of $\mathcal{SHOIN}^+_{Var}$. Then to prove: for all $(\mathcal{I}'_1|_{\sigma_{\mathcal{S}^+}}, (serv, \rho'_{in}|_{\sigma_{\mathcal{S}^+_{Var}}}), \mathcal{I}_2, \rho_{out}) \in \delta$ it holds that $(\mathcal{I}'_1|_{\sigma_{\mathcal{S}^+}}, \mathcal{I}_2, \rho'_{in,out}|_{\sigma_{\mathcal{S}^+_{Var}}}) \models_{\mathcal{S}^{+bi}_{Var}, \langle \Sigma, X_{in,out} \rangle} \mathsf{post}$, where $\rho'_{in,out}|_{\sigma_{\mathcal{S}^+_{Var}}}$ is the merging of $\rho'_{in}|_{\sigma_{\mathcal{S}^+_{Var}}}$ and ρ_{out} in the obvious way.

Let $\mathcal{I}_2 \in Q$ and $\rho_{out} \in \mathit{Output}$ such that $(\mathcal{I}'_1|_{\sigma_{\mathcal{S}^+}}, (serv, \rho'_{in}|_{\sigma_{\mathcal{S}^+_{Var}}}), \mathcal{I}_2, \rho_{out}) \in \delta$. Then there are $\mathcal{I}'_2 \in Q'$ and $\rho'_{out} \in \mathit{Output}$ such that $(\mathcal{I}'_1, (serv, \rho'_{in}), \mathcal{I}'_2, \rho'_{out}) \in \delta'$ with $\mathcal{I}_2 = \mathcal{I}'_2|_{\sigma_{\mathcal{S}^+}}$ and $\rho_{out} = \rho'_{out}|_{\sigma_{\mathcal{S}^+_{Var}}}$, by Definition 19 (reduct). We have $(\mathcal{I}'_1, \mathcal{I}'_2, \rho'_{in,out}) \models_{\mathcal{S}^{+bi}_{Var}, \langle \Sigma', X'_{in,out}\rangle} \sigma_{\mathcal{S}^{+bi}_{Var}}(\mathsf{post})$, since the precondition holds in $\mathcal{I}'_1$. By the satisfaction condition of $\mathcal{SHOIN}^{+bi}_{Var}$, we then have $(\mathcal{I}'_1|_{\sigma_{\mathcal{S}^+}}, \mathcal{I}_2, \rho'_{in,out}|_{\sigma_{\mathcal{S}^+_{Var}}}) \models_{\mathcal{S}^{+bi}_{Var}, \langle \Sigma, X_{in,out}\rangle} \mathsf{post}$.

($\Rightarrow$) Assume $\mathcal{T}'|_{\sigma_{SP}} \models_{SP, (\langle \Sigma, \Omega\rangle, Servs)} \langle serv, \mathsf{pre}, \mathsf{post}\rangle$ $(*)$. Let $\mathcal{I}_1 \in Q$, let $\rho_{in} : X_{in} \to \Delta^{\mathcal{I}_1}$, and let $(\mathcal{I}_1, \rho_{in}) \models_{\mathcal{S}^+_{Var}, \langle \Sigma, X_{in}\rangle} \mathsf{pre}$. Then, by Definition 19 and $(*)$, we have for all $(\mathcal{I}_1, (serv, \rho_{in}), \mathcal{I}_2, \rho_{out}) \in \delta$: $(\mathcal{I}_1, \mathcal{I}_2, \rho_{in,out}) \models_{\mathcal{S}^{+bi}_{Var}, \langle \Sigma, X_{in,out}\rangle} \mathsf{post}$. Let $\mathcal{I}_1 = \mathcal{I}'_1|_{\sigma_{\mathcal{S}^+}}$ and $\rho_{in} = \rho'_{in}|_{\sigma_{\mathcal{S}^+_{Var}}}$. We have $(\mathcal{I}'_1, \rho'_{in}) \models_{\mathcal{S}^+_{Var}, \langle \Sigma', X'_{in}\rangle} \sigma_{\mathcal{S}^{+bi}_{Var}}(\mathsf{pre})$, using the satisfaction condition of $\mathcal{SHOIN}^{+}_{Var}$. Then to prove: for all $(\mathcal{I}'_1, (serv, \rho'_{in}), \mathcal{I}'_2, \rho'_{out}) \in \delta'$ it holds that $(\mathcal{I}'_1, \mathcal{I}'_2, \rho'_{in,out}) \models_{\mathcal{S}^{+bi}_{Var}, \langle \Sigma', X'_{in,out}\rangle} \sigma_{\mathcal{S}^{+bi}_{Var}}(\mathsf{post})$, where $\rho'_{in,out}$ is the merging of $\rho'_{in}|_{\sigma_{\mathcal{S}^+_{Var}}}$ and ρ_{out} in the obvious way.

Let $(\mathcal{I}'_1, (serv, \rho'_{in}), \mathcal{I}'_2, \rho'_{out}) \in \delta'$. Then there are $\mathcal{I}_2 \in Q$ and $\rho_{out} \in \mathit{Output}$ such that $(\mathcal{I}_1, (serv, \rho_{in}), \mathcal{I}_2, \rho_{out}) \in \delta$ with $\mathcal{I}_2 = \mathcal{I}'_2|_{\sigma_{\mathcal{S}^+}}$ and $\rho_{out} = \rho'_{out}|_{\sigma_{\mathcal{S}^+_{Var}}}$, by Definition 19 (reduct). We have $(\mathcal{I}_1, \mathcal{I}_2, \rho_{in,out}) \models_{\mathcal{S}^{+bi}_{Var}, \langle \Sigma, X_{in,out}\rangle} \mathsf{post}$, since the precondition holds in $\mathcal{I}_1$. By the satisfaction condition of $\mathcal{SHOIN}^{+bi}_{Var}$, we then have $(\mathcal{I}'_1, \mathcal{I}'_2, \rho'_{in,out}) \models_{\mathcal{S}^{+bi}_{Var}, \langle \Sigma', X'_{in,out}\rangle} \sigma_{\mathcal{S}^{+bi}_{Var}}(\mathsf{post})$. □

Definition 20 *(service package specification)* A service package specification is a presentation $\langle \Sigma_{SP}, \Psi_{SP}\rangle$ where $\Sigma_{SP} \in |Sig_{SP}|$ and $\Psi_{SP} \subseteq Sen_{SP}(\Sigma_{SP})$ such that for each $serv \in Servs$ where $\Sigma_{SP} = \langle Ont, Servs\rangle$ there is exactly one sentence of the form $\langle serv, \mathsf{pre}, \mathsf{post}\rangle$ in Ψ_{SP}. Its semantics is the class of Σ_{SP}-models satisfying the axioms in Ψ_{SP}, i.e., $\mathbf{Mod}_{SP}(\langle \Sigma_{SP}, \Psi_{SP}\rangle)$.

D Matching Service Requests and Service Providers

Definition 21 *(matching)* Let $\langle \Sigma^R_{SP}, \Psi^R_{SP}\rangle$ and $\langle \Sigma^P_{SP}, \Psi^P_{SP}\rangle$ be service package specifications of request and provider, respectively, where $\sigma_{SP} : \Sigma^R_{SP} \to \Sigma^P_{SP}$ is an SP signature morphism. Then, the request is matched by the provider under σ_{SP} iff

$$\mathbf{Mod}_{SP}(\langle \Sigma^P_{SP}, \Psi^P_{SP}\rangle)|_{\sigma_{SP}} \subseteq \mathbf{Mod}_{SP}(\langle \Sigma^R_{SP}, \Psi^R_{SP}\rangle).$$

Theorem 2 *(characterization of matching by semantic entailment)* Let $\langle \Sigma^R_{SP}, \Psi^R_{SP}\rangle$ and $\langle \Sigma^P_{SP}, \Psi^P_{SP}\rangle$ be service package specifications of request and provider, respectively, where $\langle \Sigma^P_{SP}, \Psi^P_{SP}\rangle$ is consistent, i.e., $\mathbf{Mod}_{SP}(\langle \Sigma^P_{SP}, \Psi^P_{SP}\rangle) \neq \emptyset$, and where $\sigma_{SP} : \Sigma^R_{SP} \to \Sigma^P_{SP}$ is an SP signature morphism. Then, the request is matched by the provider under σ_{SP} according to Definition 21, if the following holds.

Let $\Sigma^R_{SP} = (\langle \Sigma^R, \Omega^R\rangle, Servs^R)$ and $\Sigma^P_{SP} = (\langle \Sigma^P, \Omega^P\rangle, Servs^P)$. Then for all $\langle serv^R, \mathsf{pre}^R, \mathsf{post}^R\rangle \in \Psi^R_{SP}$ two conditions hold for $\langle serv^P, \mathsf{pre}^P, \mathsf{post}^P\rangle \in \Psi^P_{SP}$, where $serv^P = \sigma_{SP}(serv^R)$, $\sigma_{\mathcal{S}^+} : \Sigma^R \to \Sigma^P$, $X_{in} = var_{in}(serv^P)$ and $X_{in,out} = var_{in}(serv^P) \cup var_{out}(serv^P)$:[4]

1. $\sigma_{\mathcal{S}^+}(\mathsf{pre}^R) \cup \Omega^P \models_{\mathcal{S}^+,\Sigma^P_{X_{in}}} \mathsf{pre}^P$
2. $\sigma_{\mathcal{S}^+}(\mathsf{pre}^R)@pre \cup \Omega^P@pre \cup \mathsf{post}^P \cup \Omega^P \models_{\mathcal{S}^+,\Sigma^P_{X_{in,out}}{}^{bi}} \sigma_{\mathcal{S}^+}(\mathsf{post}^R)$

The sentences $\Omega^P@pre$ are obtained from Ω^P by adding $@pre$ to all concept names and role names, and similarly for $\sigma_{\mathcal{S}^+}(\mathsf{pre}^R)@pre$.

Proof: Let $\langle \Sigma^R_{SP}, \Psi^R_{SP}\rangle$ and $\langle \Sigma^P_{SP}, \Psi^P_{SP}\rangle$ be service package specifications of request and provider, respectively, where $\langle \Sigma^P_{SP}, \Psi^P_{SP}\rangle$ is consistent, i.e., $\mathbf{Mod}_{SP}(\langle \Sigma^P_{SP}, \Psi^P_{SP}\rangle) \neq \emptyset$, and where $\Sigma^R_{SP} = (\langle \Sigma^R, \Omega^R\rangle, Servs^R)$ and $\Sigma^P_{SP} = (\langle \Sigma^P, \Omega^P\rangle, Servs^P)$. For simplicity, we assume that $\Psi^R_{SP} = \{\langle serv^R, \mathsf{pre}^R, \mathsf{post}^R\rangle\}$ and $\Psi^P_{SP} = \{\langle serv^P, \mathsf{pre}^P, \mathsf{post}^P\rangle\}$ and that $serv^R = serv^P$, denoted by $serv$, with $\sigma_{SP} : serv^R \to serv^P$. The proof can easily be extended to the general case. Let $X_{in} = var_{in}(serv^P)$ and $X_{in,out} = var_{in}(serv^P) \cup var_{out}(serv^P)$.

Assume that the two conditions of the characterization of Theorem 2 hold for $\langle serv, \mathsf{pre}^R, \mathsf{post}^R\rangle$ and $\langle serv, \mathsf{pre}^P, \mathsf{post}^P\rangle$. Then we have to show that $\mathbf{Mod}_{SP}(\langle \Sigma^P_{SP}, \Psi^P_{SP}\rangle)|_{\sigma_{SP}} \subseteq \mathbf{Mod}_{SP}(\langle \Sigma^R_{SP}, \Psi^R_{SP}\rangle)$.

Let $\mathcal{T} = \langle Q, \delta\rangle \in \mathbf{Mod}_{SP}(\langle \Sigma^P_{SP}, \Psi^P_{SP}\rangle)$. We have that $\mathcal{T} \models_{SP,\Sigma^P_{SP}} \langle serv, \mathsf{pre}^P, \mathsf{post}^P\rangle$, and must show that $\mathcal{T}|_{\sigma_{SP}} \models_{SP,\Sigma^R_{SP}} \langle serv, \mathsf{pre}^R, \mathsf{post}^R\rangle$. That is, we have to show that for all $\mathcal{I} \in Q|_{\sigma_{Ont}}$ and for all $\rho_{in} : X_{in} \to \Delta^{\mathcal{I}}$ such that $(\mathcal{I}, \rho_{in}) \models_{\mathcal{S}^+_{Var},\langle \Sigma^R, X_{in}\rangle} \mathsf{pre}^R$ the following holds:

$$\begin{aligned}&\text{for all } (\mathcal{I}, (serv, \rho_{in}), \mathcal{I}', \rho_{out}) \in \delta|_{\sigma_{SP}}\\ &\qquad\qquad\text{it holds that } (\mathcal{I}, \mathcal{I}', \rho_{in,out}) \models_{\mathcal{S}^{+bi}_{Var},\langle \Sigma^R, X_{in,out}\rangle} \mathsf{post}^R. \quad (1)\end{aligned}$$

Let $\mathcal{I} \in Q|_{\sigma_{Ont}}$ and let $\rho_{in} : X_{in} \to \Delta^{\mathcal{I}}$ be a valuation such that $(\mathcal{I}, \rho_{in}) \models_{\mathcal{S}^+_{Var},\langle \Sigma^R, X_{in}\rangle} \mathsf{pre}^R$. We then have that there is a $\mathcal{I}^u \in Q$ such that $\mathcal{I}^u|_{\sigma_{Ont}} = \mathcal{I}$, and since $\mathcal{SHOIN}^+_{Var}$ is an institution we get that $(\mathcal{I}^u, \rho_{in}) \models_{\mathcal{S}^+_{Var},\langle \Sigma^P, X_{in}\rangle} \sigma_{\mathcal{S}^+_{Var}}(\mathsf{pre}^R)$. By Definition 16 we get that $\mathcal{I}^u_{\rho_{in}} \models_{\mathcal{S}^+,\Sigma^R_{X_{in}}} \sigma_{\mathcal{S}^+_{Var}}(\mathsf{pre}^R)$ (i). Since $\mathcal{I}^u \in Q = \mathbf{Mod}_{\mathcal{S}^+}(\langle \Sigma^P, \Omega^P\rangle)$ it holds that $\mathcal{I}^u \models_{\mathcal{S}^+_{Var},\Sigma^P} \Omega^P$. By construction of $\mathcal{I}^u_{\rho_{in}}$ (which leaves the interpretation of concept names, role names, and individual names of Σ unchanged when constructing $\mathcal{I}^u_{\rho_{in}}$ from $\mathcal{I}^u$), we then have $\mathcal{I}^u_{\rho_{in}} \models_{\mathcal{S}^+,\Sigma^P_{X_{in}}} \Omega^P$ [5]. Because of (i), we then have $\mathcal{I}^u_{\rho_{in}} \models_{\mathcal{S}^+,\Sigma^P_{X_{in}}} \sigma_{\mathcal{S}^+_{Var}}(\mathsf{pre}^R) \cup \Omega^P$. Because we have $\sigma_{\mathcal{S}^+_{Var}}(\mathsf{pre}^R) \cup \Omega^P \models_{\mathcal{S}^+,\Sigma^P_{X_{in}}} \mathsf{pre}^P$ by assuming that the two conditions of the characterization of Theorem 2 hold, we have $\mathcal{I}^u_{\rho_{in}} \models_{\mathcal{S}^+,\Sigma_{X_{in}}} \mathsf{pre}^P$, and therefore $(\mathcal{I}^u, \rho_{in}) \models_{\mathcal{S}^+_{Var},\langle \Sigma^P, X_{in}\rangle} \mathsf{pre}^P$.

---

[4] We use $\sigma_{\mathcal{S}^+}(\Omega)$ as a shorthand notation for $Sen_{\mathcal{S}^+}(\sigma_{\mathcal{S}^+})(\Omega)$.

[5] Note that since $Sen_{\mathcal{S}^+}(\Sigma^P) \subseteq Sen_{\mathcal{S}^+_{Var}}(\langle \Sigma^P, X_{in}\rangle)$, and since $\Omega^P \subseteq Sen_{\mathcal{S}^+}(\Sigma^P)$, we also have $\Omega^P \subseteq Sen_{\mathcal{S}^+_{Var}}(\langle \Sigma^P, X_{in}\rangle)$.

Now, we can prove (1). Let $\mathcal{I}', \rho_{out}$ be such that $(\mathcal{I}, (serv, \rho_{in}), \mathcal{I}', \rho_{out}) \in \delta|_{\sigma_{SP}}$. Then, by the definition of $\delta|_{\sigma_{SP}}$, there must be $\mathcal{I}'^u$ such that $\mathcal{I}'^u|_{\sigma_{Ont}} = \mathcal{I}'$. Since $\mathcal{T} \in \mathbf{Mod}_{SP}(\langle \Sigma^P_{SP}, \Psi^P_{SP} \rangle)$, i.e. $\mathcal{T}$ is a model of P, it holds that $(\mathcal{I}^u, \mathcal{I}'^u, \rho) \models_{\mathcal{S}^{+bi}_{Var}, \langle \Sigma^P, X_{in,out} \rangle} \mathsf{post}^P$, where $\rho = \rho_{in,out}$. Therefore, $\overset{\curlywedge}{\mathcal{I}}{}^u_\rho \models_{\mathcal{S}^+, \Sigma^P_{X_{in,out}}{}^{bi}} \mathsf{post}^P$ by Definition 17. We also have $\overset{\curlywedge}{\mathcal{I}}{}^u_\rho \models_{\mathcal{S}^+, \Sigma^P_{X_{in,out}}{}^{bi}} \sigma_{\mathcal{S}^+_{Var}}(\mathsf{pre}^R)@pre$ by (i), and $\overset{\curlywedge}{\mathcal{I}}{}^u_\rho \models_{\mathcal{S}^+, \Sigma^P_{X_{in,out}}{}^{bi}} \Omega^P \cup \Omega^P@pre$ since $\mathcal{I}^u, \mathcal{I}'^u \in Q$.

By assuming that the two conditions of the characterization of Theorem 2 hold, we have $\sigma_{\mathcal{S}^+_{Var}}(\mathsf{pre}^R)@pre \cup \mathsf{post}^P \cup \Omega^P \cup \Omega^P@pre \models_{\mathcal{S}^+, \Sigma^P_{X_{in,out}}{}^{bi}} \sigma_{\mathcal{S}^+_{Var}}(\mathsf{post}^R)$. Therefore, we have $\overset{\curlywedge}{\mathcal{I}}{}^u_\rho \models_{\mathcal{S}^+, \Sigma^P_{X_{in,out}}{}^{bi}} \sigma_{\mathcal{S}^{+bi}_{Var}}(\mathsf{post}^R)$, and thus $(\mathcal{I}^u, \mathcal{I}'^u, \rho) \models_{\mathcal{S}^{+bi}_{Var}, \langle \Sigma^P, X_{in,out} \rangle} \sigma_{\mathcal{S}^+_{Var}}(\mathsf{post}^R)$. Using the fact that $\mathcal{SHOIN}^{+bi}_{Var}$ is an institution finally yields that $(\mathcal{I}^u|_{\sigma_{Ont}}, \mathcal{I}'^u|_{\sigma_{Ont}}, \rho) \models_{\mathcal{S}^{+bi}_{Var}, \langle \Sigma^R, X_{in,out} \rangle} \mathsf{post}^R$. This concludes the proof of (1), and with that also of $(\Leftarrow)$.

□

E Example: Garage Appointment Service

We model an appointment making service for car garages, GA. The appointment service allows to make an appointment with a garage within a given day interval, and returns an appointment. To specify the service package $\langle \Sigma^{\mathsf{GA}}_{SP}, \Psi^{\mathsf{GA}}_{SP} \rangle$, we need to provide the ontology $\langle \Sigma^{\mathsf{GA}}, \Omega^{\mathsf{GA}} \rangle$, the set of services, $Servs^{\mathsf{GA}}$, and the service specifications $\Omega^{\mathsf{GA}}_{SP}$.
Σ^{GA} is defined as follows: $N_C = \{\text{Appointment}, \text{WDay}, \text{WEDay}, \text{Day}, \text{Hour}, \text{String}\}$, $N_r = \{\text{after}, \text{ before}, \text{hasDay}, \text{hasHour}\}$, $N_i = \{1, 2, \ldots, 24, \text{mon}, \text{tue}, \ldots, \text{sun}\}$. The ontology Ω^{GA} is:

$$
\begin{aligned}
\{\, & \exists \text{hasDay.Day} \sqsubseteq \text{Appointment}, \geq 2 \text{ hasDay} \sqsubseteq \bot, \leq 0 \text{ hasDay} \sqsubseteq \bot, \\
& \exists \text{hasHour.Hour} \sqsubseteq \text{Appointment}, \geq 2 \text{ hasHour} \sqsubseteq \bot, \leq 0 \text{ hasHour} \sqsubseteq \bot, \\
& \exists \neg \text{Appointment}, \text{WDay} \sqcup \text{WEDay} \equiv \text{Day}, \\
& \text{monday} : \text{Date}, \ldots, \text{sunday} : \text{Date}, 1 : \text{Hour}, \ldots, 24 : \text{Hour}, \\
& \text{after}(\text{mon}, \text{mon}), \text{after}(\text{mon}, \text{tue}), \ldots, \\
& \text{after}(1, 1), \text{after}(1, 2), \text{after}(2, 2), \text{after}(1, 3), \text{after}(2, 3) \ldots, \\
& \text{before}(\text{mon}, \text{mon}), \text{before}(\text{tue}, \text{mon}), \ldots \}
\end{aligned}
$$

The services offered, $Servs^{\mathsf{GA}}$ are $\{makeAppointment(name, from, to) : app\}$. The service specifications Ψ^{GA}_{SP} are:

$$
\begin{aligned}
& \text{makeAppointment}(\text{String } name, \text{WDay } from, \text{WDay } to) : \text{Appointment } app \\
& \quad \mathsf{pre} \;\; \text{after}(from, to) \\
& \quad \mathsf{post} \; \text{Appointment} \sqcap \neg(\text{Appointment}@pre) \equiv \{app\}, \\
& \qquad\quad app : \exists \text{hasDay}.(\exists \text{after}.\{from\}),\ app : \exists \text{hasDay}.(\exists \text{before}.\{to\}), \\
& \qquad\quad app : \exists \text{hasHour}.(\exists \text{after}.\{8\}), \quad app : \exists \text{hasHour}.(\exists \text{before}.\{16\})
\end{aligned}
$$

For the service requester CA, a car requesting a garage appointment, we must specify the same entities: the ontology $\langle \Sigma^{\mathsf{CA}}, \Omega^{\mathsf{CA}} \rangle$, the set of requested services, $Servs^{\mathsf{CA}}$, and the service specifications $\Omega^{\mathsf{CA}}_{SP}$. Σ^{CA} is defined by $N_C = \{\text{Termin}, \text{Tag}, \text{Zeichenkette}\}$, $N_r = \{\text{nach}, \text{vor}, \text{hatTag}\}$,
$N_i = \{1, 2, \ldots, 24, \text{montag}, \text{dienstag}, \ldots, \text{sonntag}, t_1, t_2, \ldots\}$. Ω^{CA} is:

$$\begin{aligned}\{\ &\exists\text{hatTag.Tag} \sqsubseteq \text{Termin}, \geq 2\ \text{hatTag} \sqsubseteq \bot, \leq 0\ \text{hatTag} \sqsubseteq \bot,\\ &\text{montag} : \text{Tag}, \text{dienstag} : \text{Tag}, \ldots, \text{sonntag} : \text{Tag},\\ &\text{nach}(\text{montag}, \text{montag}), \text{after}(\text{montag}, \text{dienstag}), \ldots,\\ &\text{nach}(1,1), \text{nach}(1,2), \text{nach}(2,2), \text{nach}(1,3), \text{nach}(2,3) \ldots,\\ &\text{vor}(\text{montag}, \text{montag}), \text{vor}(\text{dienstag}, \text{montag}), \ldots\}\end{aligned}$$

The services required, $Servs^{\mathsf{CA}}$ are $\{terminVereinbaren(name, von, bis) : ter\}$. The service specifications Ψ^{CA}_{SP} are:

$$\begin{aligned}&\text{terminVereinbaren}(\text{Zeichenkette } name, \text{Tag } von, \text{Tag } bis) : \text{Termin } ter\\ &\quad \mathsf{pre}\ \ \text{nach}(\text{dienstag}, \text{von}), \text{nach}(\text{bis}, \text{dienstag})\\ &\quad \mathsf{post}\ \text{hatTag}(ter, \text{dienstag})\end{aligned}$$

In order to determine whether the service request CA is matched by the service provider GA, we need to define a signature morphism $\sigma : \Sigma^{\mathsf{CA}}_{SP} \rightarrow \Sigma^{\mathsf{GA}}_{SP}$. This is obtained by mapping the German notions of Σ^{CA} to the corresponding English ones of Σ^{GA}, i.e., *Termin,Tag,Zeichenkette* are mapped to *Appointment,Day,String*, respectively, *nach,vor,hatTag* are mapped to *after,before,hasDay*, respectively, and *montag,dienstag,...,sonntag* are mapped to *mon,tue,...,sun*, respectively, and *terminVereinbaren* is mapped to *makeAppointment*. The request thus specifies a service that makes an appointment on Tuesday if *from* and *to* are set to Tuesday, but it does not matter at what time.

σ yields a theory morphism from Ω^{CA} to Ω^{GA}, since the sentences of the car ontology, translated with the signature morphism σ, are a subset of the sentences of the garage ontology.

To prove that a matching between $\Sigma^{\mathsf{CA}}_{SP}$ and $\Sigma^{\mathsf{GA}}_{SP}$ exists, it must be shown that

$$\begin{aligned}&\{\text{after}(\text{tue}, \mathit{from}), \text{after}(\mathit{to}, \text{tue})\} \cup \Omega^{\mathsf{GA}}\\ &\qquad\qquad \models_{\mathcal{S}^+, \Sigma^{\mathsf{GA}}_{X_{in}}} \{\text{after}(\mathit{from}, \mathit{to}), \text{from} : \text{WDay}, \text{to} : \text{WDay}\}\end{aligned}$$

By the definition of the role "after" and since tue : WDay, it is easy to see that can be proved. Considering the postconditions, it must be shown that

$$\begin{aligned}&\{\text{after@}\mathit{pre}(\text{tuesday}, \mathit{from}), \text{after@}\mathit{pre}(\mathit{to}, \text{tuesday})\} \cup\\ &\{\text{Appointment} \sqcap \neg(\text{Appointment@}\mathit{pre}) \equiv \{\mathit{app}\},\\ &\mathit{app} : \exists\text{hasDay.}(\exists\text{after.}\{\mathit{from}\}), \mathit{app} : \exists\text{hasDay.}(\exists\text{before.}\{\mathit{to}\}),\\ &\mathit{app} : \exists\text{hasHour.}(\exists\text{after.}\{8\}), \mathit{app} : \exists\text{hasHour.}(\exists\text{before.}\{16\})\}\\ &\Omega^{\mathsf{GA}}\text{@}\mathit{pre} \cup \Omega^{\mathsf{GA}}\\ &\quad \models_{\mathcal{S}^+, \Sigma^{\mathsf{GA}\ bi}_{X_{in}, out}} \{\text{hasDay}(\mathit{app}, \text{tuesday})\}\end{aligned}$$

Removing the irrelevant sentences and keeping $\Omega^{\mathsf{GA}}@pre, \Omega^{\mathsf{GA}}$ in mind, we must prove that

$$\begin{aligned}&\{\text{after}@pre(\text{tuesday}, \mathit{from}), \text{after}@pre(\mathit{to}, \text{tuesday}),\\ &\mathit{app} : \exists\text{hasDay}.(\exists\text{after}.\{\mathit{from}\}), \mathit{app} : \exists\text{hasDay}.(\exists\text{before}.\{\mathit{to}\}),\\ &\quad \models_{\mathcal{S}^+, \Sigma^{\mathsf{GA}\ bi}_{X_{in}, out}} \{\text{hasDay}(\mathit{app}, \text{tuesday})\}\end{aligned}$$

Since $\{\mathit{app} : \exists\text{hasDay}.(\exists\text{after}.\{\mathit{from}\}),\ \mathit{app} : \exists\text{hasDay}.(\exists\text{before}.\{\mathit{to}\}),$ $\geq 2\ \text{hasDay} \sqsubseteq \bot,\ \leq 0\ \text{hasDay} \sqsubseteq \bot\}$ is given, it can be deduced that app has exaclty one day between $from$ and to. With the help of the precondition of CA, $\{\text{after}@pre(\text{tuesday}, \text{from}),\ \text{after}@pre(\text{to}, \text{tuesday})\}$, it can then be inferred that this one day must be after and before tuesday. Since there is only one day satisfying these constraints, namely tuesday, it can be inferred that $\text{hasDay}(\mathit{app}, \text{tuesday})$ holds. Hence, the service provider GA matches service request CA.